

# Moorlands Junior School



## Calculation Policy

September 2018

## **Introduction**

This document is a statement of the aims, principles and approaches for teaching and learning of calculation strategies in mathematics at Moorlands Junior School. This policy shows the mental calculation strategies and the end of year calculation expectations for each year group in each operation (addition, subtraction, multiplication and division. All mathematics should be taught with understanding rather than by rote and put into real life contexts. At the end of the policy is a glossary of terms taken from the NCETM Glossary (January 2014).

## **Rationale**

At Moorlands Junior School, we follow the Concrete-Pictorial-Abstract (CPA) approach based on Jerome Bruner's conception of the enactive, iconic and symbolic modes of representation. This approach underpins a multi-sensory approach to children's mathematical learning and supports the children to structure their experiences and refine their mathematical thinking whilst enabling every child to develop a mathematical voice.

We believe a clear progression in calculation will support the learning and teaching of mathematics throughout the school, allow clarity and provide a secure foundation upon which to build and develop mathematical skills. This has been developed in conjunction with a Mathematics Consultant and the Moorlands teaching staff; it embraces the content of National Curriculum 2014.

This policy contains the key mental and written procedures that will be taught within Moorlands Junior School. Although by the end of year 6 children should be proficient in written methods for all calculations, at the heart of all written methods is an element of mental processing. Sharing concrete and pictorial representations alongside written methods with the teacher and their peers encourages children to think about the mental strategies that underpin them and develop new ideas. Therefore, the concrete-pictorial-abstract approach helps children to prove, extend and clarify their thinking.

## **Progression in calculation should include:**

- A range of mental strategies to be used as a first resort, even when written methods have been introduced and embedded
- An ability to understand and use the relationships between the four operations of number
- An ability to explain, describe, prove and record their methods
- An ability to know whether their answer is reasonable, to value estimation and check whether the answer is correct
- An ability to solve a wide range of problems involving calculation in a variety of contexts
- An ability to choose and use the most appropriate method of calculation; mental, pictorial (jottings) or abstract (written)
- An ability to take the initiative to return to an earlier approach that children are more confident with if necessary

**Children should be encouraged to see mathematics as a mental and spoken language with concrete models, pictorial representations and abstract recording as a written 'record' of these processes. Teachers should support and guide children through the following stages:**

**Developmental Aims:**

- To introduce children to the processes of calculation through concrete and pictorial investigative activities
- To support children in developing ways of recording in order to support their thinking and mental calculation methods
- To enable children to interpret and use mathematical signs and symbols
- To facilitate children's use of concrete objects and pictorial representations, beginning with concrete resources such as Numicon shapes and moving to visual representations such as an empty number-line before progressing to abstract understanding
- To enable children to strengthen and refine their mental methods in order to develop informal written methods
- To support children in becoming more efficient and succinct in their recordings which will ultimately lead them to decide when a written method should be used
- All children should be equipped with mental and written methods that they understand and can use correctly
- All children will be able to decide which approach is the most appropriate and have strategies to check its accuracy
- At whatever stage in their learning, and whatever approach is being used, children's methods of calculating will be underpinned by a secure and appropriate knowledge of number facts, along with the mental skills that are needed to carry out the process and judge if it was successful

**The overall aims when children leave primary school are for them to:**

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that they develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately. This will include:
  - having a secure understanding of known mathematics facts to apply to written mathematics
  - having a sound knowledge of number facts and a good understanding of the four operations
  - having an efficient, reliable, compact written method of calculation for each operation that can be applied with confidence when undertaking calculations that cannot be carried out mentally
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

### **Mental methods of calculation**

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A 'feel' for number is the outcome of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of concrete objects and visual representations and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- understand
  - the different **structures** of all four operations e.g. to understand subtraction as take away, decrease, difference and the inverse;
  - the **relationship** between operations - that subtraction 'undoes' addition, how multiplication and division relate to one another
  - how the **rules and laws** of arithmetic are used and applied – e.g. to add or subtract mentally ones, tens or hundreds from a three-digit number (Year 3), and to perform mental calculations, including with mixed operations and large numbers (Year 6)
- recall key number facts instantly – building upon the strategies that will have been developed in KS1, for example, to recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 (Year 2)
- memorise the multiplication tables up to and including 12 x 12 by the end of year 4  
Developed as follows:
  - **Year 3** – Refine 2, 10, 5 and 3 times tables. Learn 11, 4 and 8 times tables
  - **Year 4**– Refine 2, 10, 5, 3, 4, 8 and 11 times tables. Learn 6, 9, 12 and 7 times tables
  - **Year 5/6** – continue to practice all times tables up to 12 x 12, use these to inform division and to work out other times tables higher than 12 (e.g. double 12 times tables to generated 24 times tables)
- use taught strategies to work out the calculation - for example, to select a mental strategy appropriate for the numbers involved in the calculation (Year 5)

### **Written methods of calculation**

The aim is that by the end of Key Stage 2, the children should be able to use an efficient written method for each operation with confidence and understanding. Children will develop the ability to use what are commonly known as 'standard' written methods - methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools that they can use when they are unable to carry out the calculation in their heads or by using jottings. We want children to know that they have such a reliable written method to which they can turn when the need arises.

In setting out these aims, the intention is that there will be a consistent approach to the learning of calculation strategies and that all teachers understand the progression of skills and key concepts. The children will benefit greatly from learning how to use the most efficient methods. The challenge for teachers will be in determining when their children should move on to a refinement in the method and become confident and more efficient at

written calculations. Guidance is given within this document in reaching the most efficient methods for each of the four number operations.

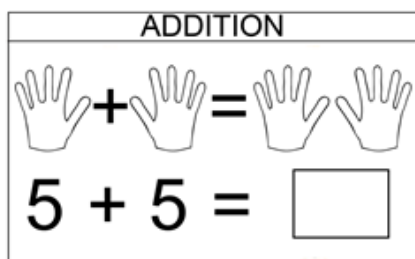
## Mathematical Language

For all calculations, we need children to communicate mathematically by practically carrying out the calculation (being active), visualising what they have done mentally and on paper (illustrate) so that they can talk about it (language). Therefore, it is extremely important that children are introduced to the mathematical language and practise using this to explain their actions, thinking and the reasoning behind the strategy they have used.

Children's early recording should show their conceptual understanding. They will need to use the mathematical language to explain their workings and understanding.

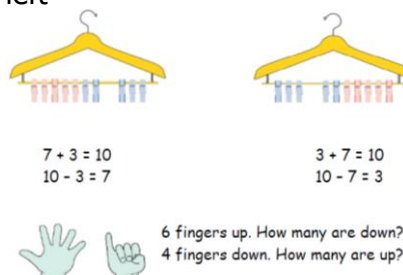
### **Addition**

Add  
And  
Plus  
Increase  
More than  
Total  
Altogether  
Sum of



### **Subtraction**

Take away  
Minus  
Decrease  
Less than  
Difference  
How much more  
Subtract  
left



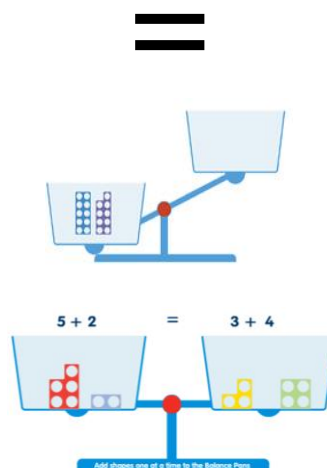
### **Multiply**

Sets of  
Lots of  
Groups of  
Times  
Multiply by  
Double – twice, three times etc.  
Product  
Multiple  
Factor  
Repeated addition

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

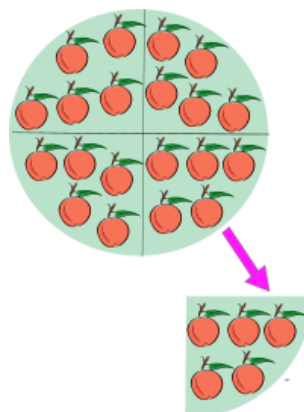
### **Equivalence**

Is the same value as . . .  
Is equivalent to . . .  
Makes  
Is equal to . . .  
Has the same value but looks different



### **Division**

Share equally  
Divide by  
Group  
Halve, quarter etc.  
Chunk  
Multiple  
Pairs  
Remainders  
Array



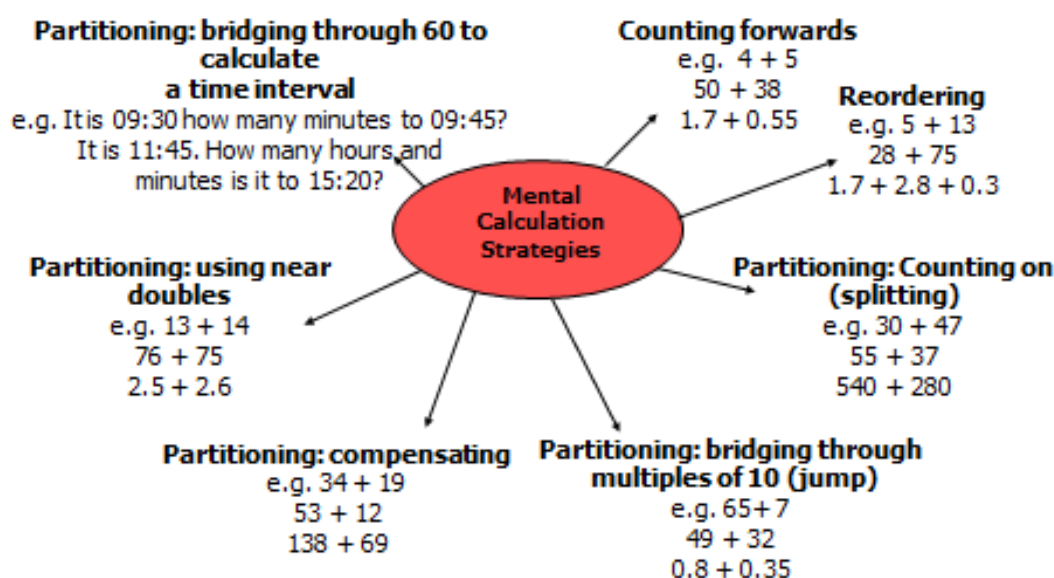
## Progression from Mental Methods to Written Methods for Addition

### **To add successfully, children need to be able to:**

- count and have an understanding of quantity including greater than and less than symbols
- recall all addition pairs to  $9 + 9$  and complements in 10;
- add mentally a series of one-digit numbers, such as  $5 + 8 + 4$ ;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways

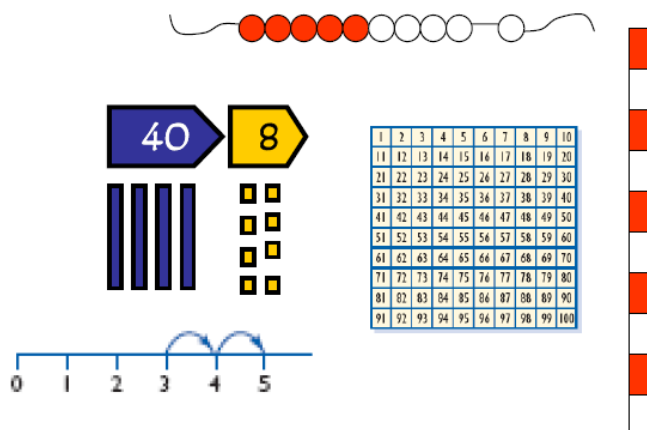
### **Mental Skills**

It is important that children's mental methods of calculation are practised and secured alongside their learning of written methods (see introduction). Below is an outline of the key addition strategies which are exemplified in each of the year group pages.



### **Concrete resources and pictorial representations**

Counting apparatus  
Place value apparatus  
Place value cards  
Number tracks  
Numbered number lines  
Marked but unnumbered number lines  
Empty number lines  
Hundred square  
Counting stick  
Bead string



## **Addition Strategies**

### **Counting forwards**

Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting forwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting forwards and the relative position of numbers.

They will also learn that, when they add two numbers together, it is generally easier to count on from the larger number rather than the smaller. You will need to review children's 'counting on' strategies, then show them and encourage them to adopt more efficient methods.

### **Reordering**

Sometimes a calculation can be more easily worked out by changing the order of the numbers. The way in which children rearrange numbers in a particular calculation will depend on which number facts they can recall or derive quickly.

It is important for children to know when numbers can be reordered, for example,  $2 + 5 + 8 = 8 + 2 + 5$  or  $15 + 8 - 5 = 15 - 5 + 8$

The strategy of changing the order of numbers applies mainly when the question is written down. It is more difficult to reorder numbers if the question is presented orally.

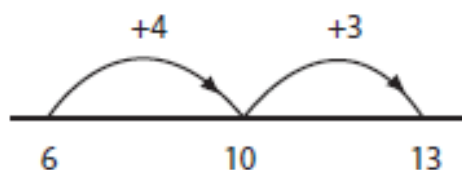
### **Regrouping: Counting on (splitting strategy)**

It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that  $326 = 300 + 20 + 6$ . In this way, numbers are seen as wholes, rather than as a collection of single digits in columns.

This way of regrouping numbers can be a useful strategy for adding pairs of numbers. Both numbers can be partitioned, *although it is often helpful to keep the first number as it is and to partition just the second number.*

### **Regrouping: bridging through multiples of 10 (jump strategy)**

An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40. In mental addition, it is often useful to count on in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them. For example,  $6 + 7$  is worked out in two jumps, first to 10, then to 13. The answer is the last point marked on the line, 13.



A similar method can be applied to decimals, but here, instead of building up to a multiple of 10, bridging is through the next whole number. So  $2.8 + 1.6$  is  $2.8 + 0.2 + 1.4 = 3 + 1.4$ .

### **Regrouping: compensation (adjustment strategy)**

This strategy is useful for adding numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be added is rounded to a multiple of 10 plus or minus a small number. For example, adding 9 is carried out by adding 10, then subtracting 1.

A similar strategy works for adding decimals that are close to whole numbers. For example:  $1.4 + 2.9 = 1.4 + 3 - 0.1$

### **Regrouping: using 'near' doubles**

If children have instant recall of doubles, they can use this information when adding two numbers that are very close to each other. So, knowing that  $6 + 6 = 12$ , they can be encouraged to use this to help them find  $7 + 6$ , rather than use a counting on strategy or bridging through 10.

### **Regrouping: bridging through 60 to calculate a time interval**

Time is a universal non-metric measure.

A digital clock displaying 9.59 will, in two minutes time, read 10.01 not 9.61. When children use minutes and hours to calculate time intervals, they have to bridge through 60.

So to find the time 20 minutes after 8.50am, for example, children might say 8.50am plus 10 minutes takes us to 9.00am, then add another 10 minutes.

## Year 3

### Counting forwards

#### **Example Calculation**

$$150 + 18$$

$$134 + 65$$

$$135 + 115$$

#### **Possible counting strategy**

Count on in tens then ones from 150

Count on in tens then ones from 134

Count on one hundred then in steps of 5 from 135

### Reordering

#### **Example Calculation**

$$23 + 154$$

$$120 + 50 + 120$$

#### **Possible reordering strategy**

$$154 + 23$$

$$120 + 120 + 50$$

### Regrouping: counting on

#### **Example Calculation**

$$123 + 12$$

$$330 + 47$$

$$117 + 114$$

#### **Possible regrouping and counting strategy**

$$123 + 10 + 2$$

$$330 + 40 + 7$$

$$117 + 100 + 10 + 4$$

### Regrouping: bridging through multiples of 10

#### **Example Calculation**

$$149 + 8$$

$$655 + 7$$

#### **Possible bridging strategy**

$$149 + 1 + 7$$

$$655 + 5 + 2$$

### Regrouping: compensation

#### **Example Calculation**


$$287 + 12$$

$$343 + 28$$

#### **Possible strategy**

$$287 + 10 + 2$$

$$343 + 30 - 2$$



Compensation is a strategy that some adults find really challenging. You may decide to discuss this with selected children

### Regrouping: using 'near' doubles

#### **Example Calculation**

$$160 + 170$$

#### **Possible compensation strategy**

is double 150, then add 10, then add 20

or double 160 and add 10

or double 170 and subtract 10

### Regrouping: bridging through 60 to calculate a time interval

#### **Example Calculation**

It is 10:30am. How many minutes to 10.45am?

It is 3:45pm. How many minutes to 4.15pm?

## Year 3

**Children should choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

### Regrouping in different ways

Revisit the Year 2 approaches at the start of the year progressing to:

$$\begin{array}{r} 243 + 56 = 299 \\ \swarrow \quad \searrow \\ 200 + 90 + 9 \end{array} \quad \text{Initially no regrouping progressing to regrouping}$$

Continue to use a range of concrete resources including Dienes, Numicon Shapes and counters. Children should progress to 3 digits.

$$537 + 48$$

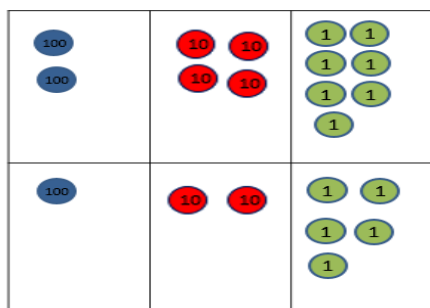
48

3

45

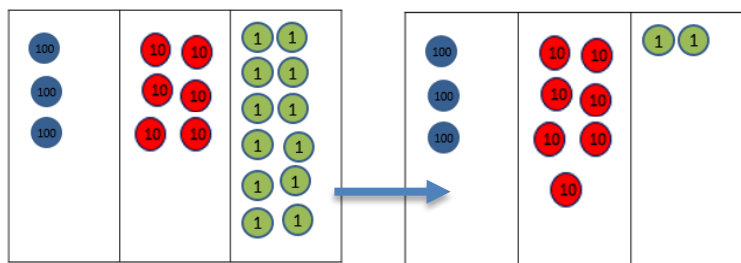
### Towards a written method

Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



$$\begin{array}{r} 200 + 40 + 7 \\ 100 + 20 + 5 \\ \hline 300 + 60 + 12 = 372 \end{array}$$

Leading to children understanding the exchange between tens and ones



$$\begin{array}{r} 247 \\ + 125 \\ \hline 372 \\ \hline 1 \end{array} \rightarrow$$

The formal method should be seen as a more streamlined version of the expanded method, not a new method.

**Missing number problems** – often set in a context or used with a **bar model**

$$140 + 150 = \square$$

$$320 + \square + 9 = 1000$$

$$521 + \square = 600$$

### Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

## **Year 4**

### **Counting forwards**

#### **Example Calculation**

$$570 + 300$$

$$47.3 + 0.6$$

#### **Possible counting strategy**

Count on in hundreds from 570

Count on in tenths

### **Reordering**

#### **Example Calculation**

$$28 + 375$$

$$1.7 + 2.8 + 0.3$$

#### **Possible reordering strategy**

$375 + 28$  (thinking of 28 as  $25 + 3$ )

$$1.7 + 0.3 + 2.8$$

### **Regrouping: counting on**

#### **Example Calculation**

$$43 + 28 + 51$$

$$5.6 + 3.7$$

#### **Possible regrouping and counting strategy**

$$40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1$$

or  $43 + 1 + 6 + 22 + 50$  or  $43 + 8 + 51 + 20$

$$5.6 + 3 + 0.7 = 8.6 + 0.7$$

### **Regrouping: bridging through multiples of 10**

#### **Example Calculation**

$$145 + 28$$

$$655 + 236$$

#### **Possible bridging strategy**

$$145 + 5 + 23$$

$$655 + 5 + 31 + 200$$

### **Regrouping: compensation**

#### **Example Calculation**

$$138 + 69$$

$$5.6 + 1.9$$

#### **Possible compensation strategy**

$$138 + 70 - 1$$

$$5.6 + 2 - 0.1$$

### **Regrouping: using 'near' doubles**

#### **Example Calculation**

$$2.5 + 2.6$$

#### **Possible compensation strategy**

is double 2.5 and add 0.1

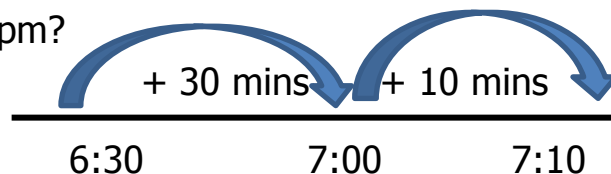
or double 2.6 and subtract 0.1

### **Regrouping: bridging through 60 to calculate a time interval**

#### **Example Calculation**

I get up 40 minutes after 6:30 am. What time is that?

It is 4:26 pm. How many minutes to 5:05 pm?



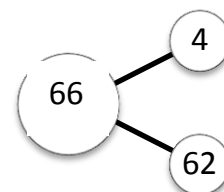
## Year 4

Children should continue to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

### Regrouping in different ways

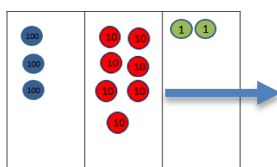
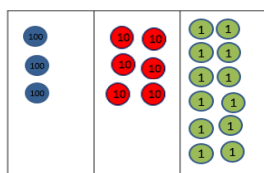
Revisit the Year 3 approaches if necessary and continue to develop the part part whole model.

$$626 + 66$$



### Written methods (progressing to 4-digits)

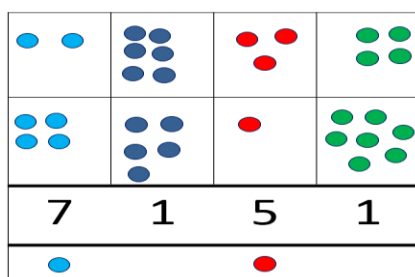
Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.



$$\begin{array}{r} 247 \\ + 125 \\ \hline 372 \\ 1 \end{array} \rightarrow$$

### Compact written method

Extend to numbers with at least four digits.



$$\begin{array}{r} 2634 \\ + 4517 \\ \hline 7151 \\ 1 \quad 1 \end{array}$$

Extend to up to two decimals places, initially with money and only when children are fully secure with whole number calculations. Continue to develop understanding with questions that have the same number of decimals places and adding several numbers (with different numbers of digits). Some children will benefit from using place value counters initially.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 1 \end{array}$$

### Missing number problems

$$6.5 + 2.7 = \square$$

$$4,087 + \square = 5,000$$

$$7.2 + \square = 9$$

## **Vocabulary**

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones / tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? equals sign, is the same as

## **Year 5**

### **Counting forwards**

#### **Example Calculation**

$4,800 + 400$   
 $47.33 + 0.69$

#### **Possible counting strategy**

Count on in hundreds from 4800  
Count on in tenths and hundredths

### **Reordering**

#### **Example Calculation**

$180 + 650$   
 $34 + 27 + 46$

#### **Possible reordering strategy**

$650 + 180$  (thinking of 180 as  $150 + 30$ )  
 $34 + 46 + 27$

### **Regrouping: counting on**

#### **Example Calculation**

$540 + 280$   
 $15.65 + 3.72$

#### **Possible regrouping and counting strategy**

$540 + 200 + 80$   
 $15.65 + 3.02 + 0.7 = 18.67 + 0.7$

### **Regrouping: bridging through multiples of 10**

#### **Example Calculation**

$1.4 + 1.7$   
 $543 + 248$

#### **Possible bridging strategy**

$1.4 + 0.6 + 1.1$   
 $543 + 7 + 41 + 200$

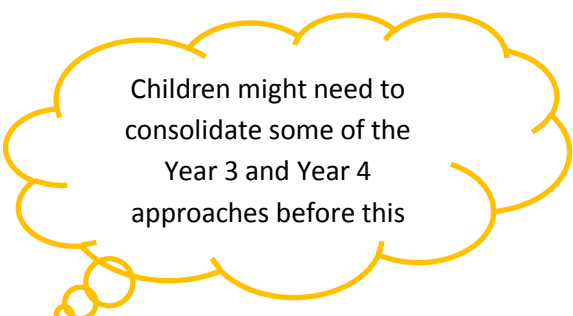
### **Regrouping: compensation**

#### **Example Calculation**

$1,308 + 690$   
 $29.6 + 11.9$

#### **Possible strategy**

$1,308 + 700 - 10$   
 $29.6 + 12 - 0.1$



Children might need to consolidate some of the Year 3 and Year 4 approaches before this

### **Regrouping: using 'near' doubles**

#### **Example Calculation**

$3,300 + 3,200$

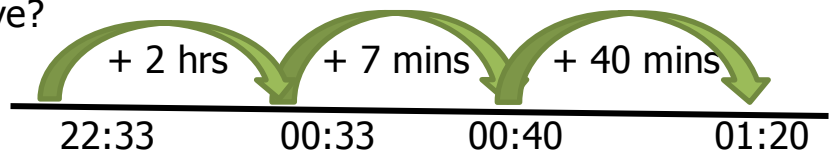
#### **Possible compensation strategy**

is double 3,200 and add 100  
or double 3,300 and subtract 100

### **Regrouping: bridging through 60 to calculate a time interval**

#### **Example Calculation**

A train leaves London for Leeds at 22:33.  
 The journey takes 2 hours and 47 minutes.  
 What time does the train arrive?



## Year 5

**Children should continue to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

### Written methods (progressing to more than 4-digits)

Children calculate using the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

$  \begin{array}{r}  172.83 \\  + 54.68 \\  \hline  227.51 \\  \text{1 1 1}  \end{array}  $	$  \begin{array}{r}  24087 \\  + 3196 \\  \hline  27283 \\  \text{1 1}  \end{array}  $
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Concrete resources can be used alongside the columnar method to develop understanding of addition with decimal numbers or when moving to larger numbers if children are not secure.

### Missing number problems

$$12.43 + 9.81 = \square \qquad 24,087 + \square = 35,000 \qquad 7.26 + \square = 9.52$$

### Vocabulary

Tens of thousands boundary *plus all previous vocabulary*

## Year 6

### Counting forwards

#### **Example Calculation**

$$512,893 + 9,000$$

$$5,128.93 + 0.37$$

#### **Possible counting strategy**

Count on in thousands

Count on in tenths and hundredths

### Reordering

#### **Example Calculation**

$$1,052 + 385,934$$

$$1.4 + 3.2 + 0.6 + 0.2$$

#### **Possible reordering strategy**

$$385,934 + 1,052$$

$$1.4 + 0.6 + 3.2 + 0.2$$

### Regrouping: counting on

#### **Example Calculation**

$$86 + 385,934$$

$$135.25\text{cm} + 4.72\text{cm}$$

#### **Possible regrouping and counting strategy**

$$385,934 + 86 \text{ (thinking of 86 as } 6 + 80)$$

$$135.25 + 4 + 0.7 + 0.02$$

### Regrouping: bridging through multiples of 10

#### **Example Calculation**

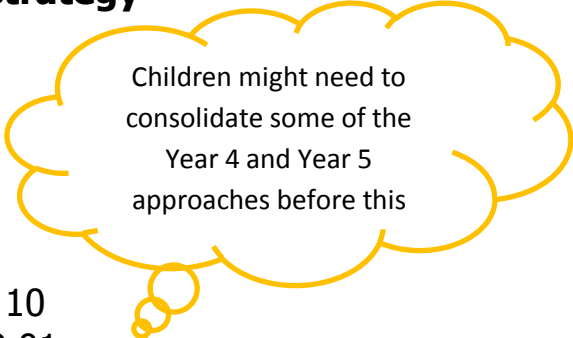
$$10.8 + 0.35$$

$$2,589 + 307$$

#### **Possible bridging strategy**

$$10.8 + 0.2 + 0.15$$

$$2,589 + 1 + 306$$



Children might need to consolidate some of the Year 4 and Year 5 approaches before this

### Regrouping: compensation

#### **Example Calculation**

$$5,893,614 + 6,090$$

$$£175.99 + £4.99$$

#### **Possible strategy**

$$5,893,614 + 6,100 - 10$$

$$£175.99 + £5.00 - £0.01$$

### Regrouping: using 'near' doubles

#### **Example Calculation**

$$4,850 + 4,750$$

#### **Possible compensation strategy**

is double 4,800

### Regrouping: bridging through 60 to calculate a time interval

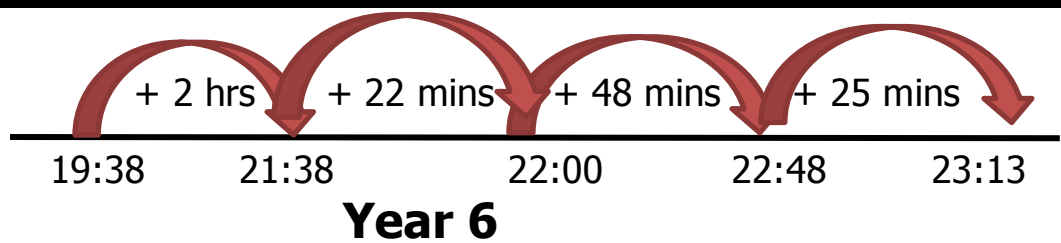
#### **Example Calculation**

A train leaves London for Leeds at 19:38.

The journey takes 2 hours and 47 minutes.

The train is delayed by 48 minutes.

What time does the train arrive?



**Children should continue to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

### **Written methods**

As year 5, progressing to larger numbers; aiming for both conceptual understanding and procedural fluency with a secure columnar method.

$$\begin{array}{r}
 93772.83 \\
 + 6454.687 \\
 \hline
 100227.517 \\
 \text{1 1 1 1 1}
 \end{array}$$

$$\begin{array}{r}
 8924087 \\
 + 4573196 \\
 \hline
 13497283 \\
 \text{1 1 1 1}
 \end{array}$$

Continue calculating with decimals, including those with different numbers of decimal places. If the children are unsure reduce the number size until they become secure.

### **Missing number problems**

$$12.43 + 19.8 = \square$$

$$23,507 + \square = 35,429$$

$$7.26 + \square = 29.5$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|c|} \hline \square & 6 & \square & 8 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline 3 & \square & 9 & \square \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|} \hline 9 & 0 & 1 & 9 \\ \hline \end{array}
 \end{array}$$

### **Vocabulary**

Consolidation of previous years

### **Progression from Mental Methods to Written Methods for Subtraction**

*To subtract successfully, children need to be able to:*

- recall all addition and subtraction facts to 20
- subtract multiples of 10 (such as  $160 - 70$ ) using the related subtraction fact,  $16 - 7$ , and their knowledge of place value
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into  $70 + 4$  or  $60 + 14$ ).

### **Mental Skills**

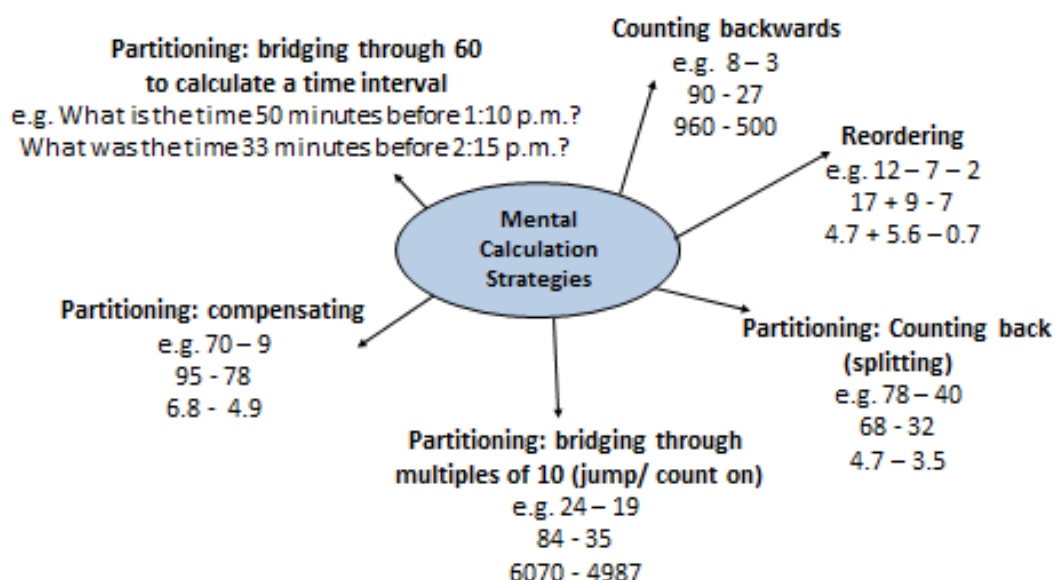
Recognise the size and position of numbers Count back in ones and tens

Know number facts for all numbers to 20. Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Bridge through 10

It is important that children's mental methods of calculation are practised and secured alongside their learning of written methods. The children should decide which is the most efficient method to use e.g. mental, jotting or written.



### **Concrete resources and pictorial representations**

Place value apparatus

Place value cards

Number tracks

Numbered number lines

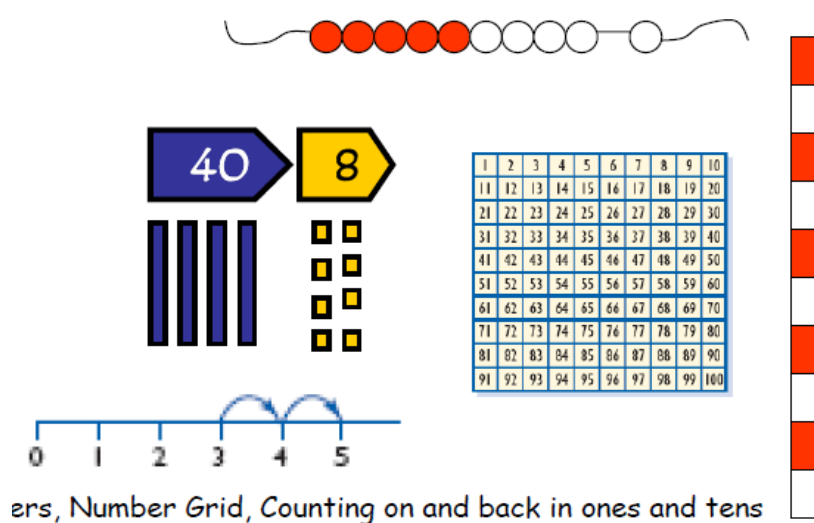
Marked but unnumbered lines

Hundred square

Empty number lines.

Counting stick

Bead strings



## Subtraction Strategies

### **Counting backwards**

Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting backwards and the relative position of numbers.

You will need to review children's 'counting back' strategies, then show them and encourage them to adopt more efficient methods. This strategy links to the 'reduction' or 'decrease' structure. If the price of a jumper costing £12.00 is reduced by £5.00, what is the new price?

### **Reordering**

Sometimes a calculation can be more easily worked out by changing the order of the numbers. The way in which children rearrange numbers in a particular calculation will depend on which number facts they can recall or derive quickly.

It is important for children to know when numbers can be reordered, for example,  $15 + 8 - 5 = 15 - 5 + 8$  or  $23 - 9 - 3 = 23 - 3 - 9$  and when they can't be reordered:  $8 - 5 \neq 5 - 8$

The strategy of changing the order of numbers applies mainly when the question is written down. It is more difficult to reorder numbers if the question is presented orally.

### **Regrouping: Counting back (splitting strategy)**

It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that  $326 = 300 + 20 + 6$ . In this way, numbers are seen as wholes, rather than as a collection of single digits in columns.

This way of regrouping numbers can be a useful strategy for subtracting pairs of numbers. Both numbers can be partitioned, *although it is often helpful to keep the first number as it is and to partition just the second number.*

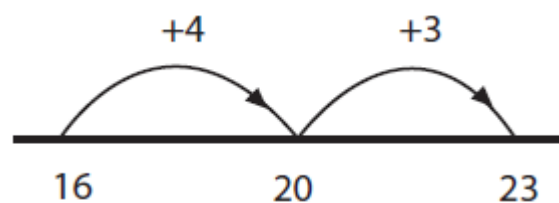
### **Regrouping: bridging through multiples of 10 (jump strategy)**

An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40.

In mental subtraction, it is often useful to count back in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them.

Subtraction, the inverse of addition, can be worked out by counting back from the larger number. But it can also be represented as the difference or 'distance' between two numbers. The distance is often found by counting up from the smaller to the larger number, again bridging through multiples of 10 or 100. This method of complimentary addition is sometimes called 'shopkeepers method' because it is like a shop assistant counting out change. So the change from £1 for a purchase of 37p is found by counting coins into the hand; '37p and 3p is 40p, and 10p makes 50p, and 50p makes £1.'

The empty number line can give an image for this method. The calculation  $23 - 16$  can be built up as an addition:



'16 and 4 is 20, and 3 is 23, so add  $4 + 3$  for the answer.' In this case the answer of 7 is not a point on the line but is the total distance between the two numbers 16 and 23.

A similar method can be applied to decimals, but here, instead of building up to a multiple of 10, bridging is through the next whole number.

### **Regrouping: compensation (adjustment strategy)**

This strategy is useful for subtracting numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be subtracted is rounded to a multiple of 10 plus or minus a small number. For example, subtracting 18 is carried out by subtracting 20, then adding 2.

A similar strategy works for subtracting decimals that are close to whole numbers. For example:  $2.45 - 1.9 = 2.45 - 2 + 0.1$

### **Regrouping: bridging through 60 to calculate a time interval**

Time is a universal non-metric measure.

When children use minutes and hours to calculate time intervals, they have to bridge through 60.

## Year 3

### Counting backwards

#### **Example Calculation**

$190 - 27$   
 $187 - 33$   
 $335 - 115$

#### **Possible counting strategy**

Count back in tens then ones from 190  
Count back in tens then ones from 187  
Count back one hundred then in steps of 5 from 235

### Reordering

#### **Example Calculation**

$12 - 7 - 2$   
 $17 + 9 - 7$

#### **Possible reordering strategy**

$12 - 2 - 7$   
 $17 - 7 + 9$  or  $9 - 7 + 17$

### Regrouping: counting back

#### **Example Calculation**

$168 - 32$   
 $365 - 40$

#### **Possible regrouping and counting strategy**

$168 - 30 - 2$   
 $365 - 40$

### Regrouping: bridging through multiples of 10 (Difference)

#### **Example Calculation**

$124 - 119$   
 $290 - 227$

#### **Possible bridging strategy**

$119 + 1 + 4$   
 $227 + 3 + 60$

### Regrouping: compensation

#### **Example Calculation**

$153 - 12$   
 $284 - 18$

#### **Possible strategy**

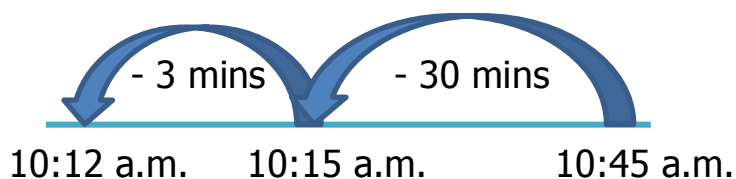
$153 - 10 - 2$   
 $284 - 20 + 2$

Compensation is a strategy that some adults find really challenging. You may decide to discuss this with selected children.

### Regrouping: bridging through 60 to calculate a time interval

#### **Example Calculation**

It is 10:45am. What was the time 33 minutes earlier?  
I get up 45 minutes before 7:30am. What time is that?



### Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

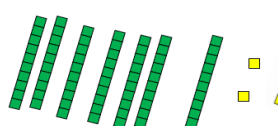
## Year 3

Children should choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

### Towards written methods

Children should have a secure understanding of the Y2 approaches and be encouraged to make choices about whether to use complementary addition (difference) or counting back (take away), depending on the numbers involved.

Recording subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus or Numicon e.g. children make the 75, they then remove 43. They then identify the answer. The children will record each step as they practically solve the calculation



– 43

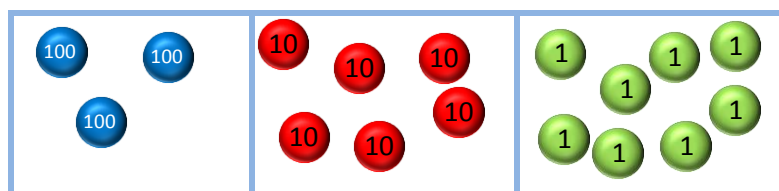
70 5

$$\begin{array}{r} - 40 \ 3 \\ \hline 30 \ 2 \end{array}$$

It is important that children experience 'exchanging' a ten into ones to support the understanding of decomposition before the more formal written methods are introduced

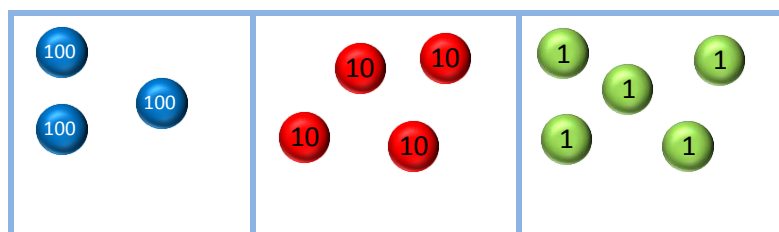
### Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition initially, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



$$\begin{array}{r} 300 \ 60 \ 8 \\ - 100 \ 40 \ 5 \\ \hline 200 \ 20 \ 3 \end{array}$$

Some children may begin to progress towards a more formal method that shows the exchange (**you should introduce this by using two digit numbers initially**). You will notice that the digits are crossed out and replaced with the exchanged values. **Progress individual children to compact method when their understanding is secure.**



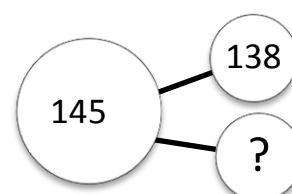
$$\begin{array}{r} 300 \ 30 \ 15 \\ - 100 \ 40 \ 5 \\ \hline 200 \ 10 \ 6 \end{array}$$

### Missing number problems

$$\square = 43 - 27$$

$$145 - \square = 138$$

$$364 - 153 = \square$$



## Year 4

### Counting backwards

#### **Example Calculation**

$$960 - 500$$

$$7.8 - 0.5$$

#### **Possible counting strategy**

Count back in hundreds from 960

Count back in tenths

### Reordering

#### **Example Calculation**

$$58 + 47 - 38$$

$$4.7 - 2.8 + 0.3$$

#### **Possible reordering strategy**

$$58 - 38 + 47$$

$$4.7 + 0.3 - 2.8$$

### Regrouping: counting back

#### **Example Calculation**

$$376 - 120$$

$$4.7 - 3.5$$

#### **Possible regrouping and counting strategy**

$$376 - 100 - 20$$

$$4.7 - 3 - 0.5$$

### Regrouping: bridging through multiples of 10 (Difference)

#### **Example Calculation**

$$607 - 288$$

$$749 - 486$$

#### **Possible bridging strategy**

$$288 + 12 + 300 + 7$$

$$486 + 14 + 200 + 49$$

### Regrouping: compensation

#### **Example Calculation**

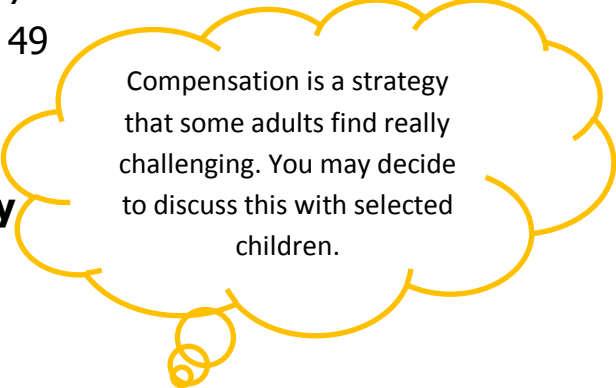
$$395 - 78$$

$$562 - 33$$

#### **Possible strategy**

$$395 - 80 + 2$$

$$532 - 30 - 3$$



Compensation is a strategy that some adults find really challenging. You may decide to discuss this with selected children.

### Regrouping: bridging through 60 to calculate a time interval

#### **Example Calculation**

What is the time 50 minutes before 13:10? (**See Year 3 number line**)

The train journey takes 48 minutes. It arrives at the station at 4:12pm, what time did it leave?

### Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? equals sign, is the same as.

## Year 4

Children should choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

**Mental methods** should continue to be developed, supported by a concrete – pictorial – abstract approach. The bar model should continue to be used to help with problem solving.

### Written methods (progressing to 4-digits)

Some children may still need to consolidate their understanding of subtraction and may need to **repeat the Year 3 approach without any exchange** (decomposition). You will notice that the digits are crossed out and replaced with the exchanged values.

	$\begin{array}{r} 300 \\ - 100 \\ \hline 200 \end{array}$
--	---

Continue to support their understanding of exchange through a double exchange worked through practically with expanded notation.

	$\begin{array}{r} 300 \\ - 100 \\ \hline 200 \end{array}$
--	---

**The formal compact method should be introduced as a more streamlined version of the expanded method, not a new method.** Children can still use place value counters to support their understanding and as with early development stages you should drop back a stage (no decomposition) to secure the understanding before moving on e.g.  $345 - 123$

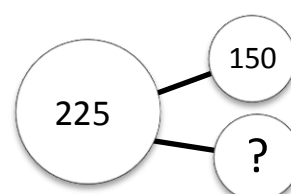
	$\begin{array}{r} 345 \\ - 129 \\ \hline 216 \end{array}$
--	---

**Extend to 4-digit number expanded column subtraction with decomposition, modelled with place value counters. Some children may progress further and be able to use the compact method.**

### Missing number problems

$$\square = 434 - 273 \quad 200 - 90 - 80 = \square$$

$$225 - \square = 150 \quad \square - 2000 = 900$$



## Year 5

### Counting backwards

#### **Example Calculation**

$4,200 - 600$   
 $47.33 - 0.65$

#### **Possible counting strategy**

Count back in hundreds from 4,200  
Count back in tenths and hundredths

### Reordering

#### **Example Calculation**

$345 + 273 - 123$   
 $4.75 + 6.48 - 3.55$

#### **Possible reordering strategy**

$273 - 123 + 345$   
 $4.75 - 3.55 + 6.48$

### Regrouping: counting back

#### **Example Calculation**

$3,893 - 1,452$   
 $18.65 - 11.32$

#### **Possible regrouping and counting strategy**

$3,893 - 1,000 - 400 - 50 - 2$   
 $18.65 - 11 - 0.3 - 0.02$

### Regrouping: bridging through multiples of 10

#### **Example Calculation**

$5.6 - 3.7$   
 $8.3 - 2.8$

#### **Possible bridging strategy**

$5.6 - 0.6 - 3 - 0.1$   
 $2.8 + 0.2 + 5.3$  or  $8.3 - 2.3 - 0.5$

### Regrouping: compensation

#### **Example Calculation**

$405 - 399$   
 $6.8 - 4.9$

#### **Possible strategy**

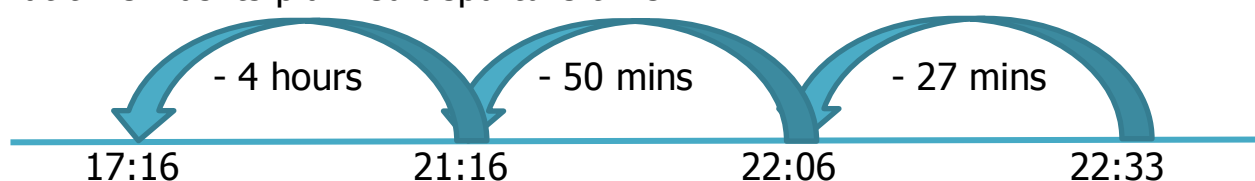
$405 - 400 + 1$   
 $6.8 - 5 + 0.1$

Compensation is a strategy that some adults find really challenging. You may decide to discuss this with selected children.

### Regrouping: bridging through 60 to calculate a time interval

#### **Example Calculation**

A train arrives in Birmingham from Edinburgh at 22:33.  
The journey takes 4 hours and 50 minutes. It was delayed for 27 minutes.  
What time was its planned departure time?



### Vocabulary

tens of thousands boundary, *also consolidation of previous years*

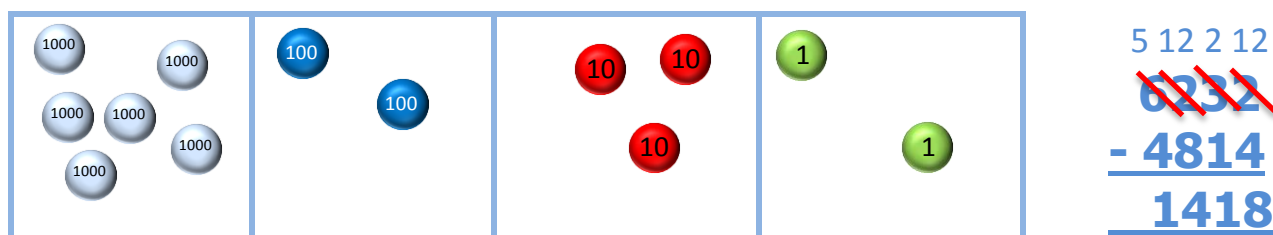
## Year 5

Children should choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

**Mental methods** should continue to be developed, supported by a concrete – pictorial – abstract approach. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency.

### Written methods (progressing to more than 4-digits)

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters. You will notice that the digits are crossed out and replaced with the exchanged values.



Progress to calculating with decimals, including those with different numbers of decimal places by using place value counters until the concept is secure.

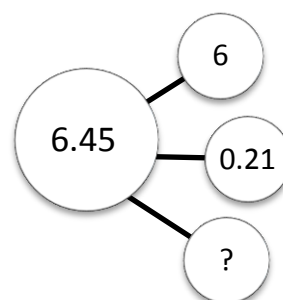
When children fully understand the use of the superscript they can progress to the more recognised form as modelled below. Extend with larger numbers.

$$\begin{array}{r} 5 \text{ } 6^{12} \text{ } 2^{2} \text{ } 3^{12} \\ \cancel{6} \cancel{3} \cancel{3} \cancel{2} \\ - 4814 \\ \hline 1418 \end{array}$$

### Missing number problems

$$1,000,000 - \square = 999,000 \quad 12,462 - 2,300 = \square$$

$$6.45 = 6 + 0.21 + \square$$



## Year 6

### Counting backwards

#### **Example Calculation**

512,893 - 9,000

128.93 - 0.37

10,000,000 - 5

#### **Possible counting strategy**

Count back in thousands

Count back in tenths and hundredths

Count back in ones or partition – 1 – 4

### Reordering

#### **Example Calculation**

1,052 + 4,523 - 325

53.78 - 12.33 - 8.18

#### **Possible reordering strategy**

4,523 - 325 + 1052

53.78 - 8.18 - 12.33

### Regrouping: counting back

#### **Example Calculation**

85,934 - 24,702

12.548 - 6.205

#### **Possible regrouping and counting strategy**

85,934 - 24,00 - 702

12.548 - 6 - 0.2 - 0.005

### Regrouping: bridging through multiples of 10

#### **Example Calculation**

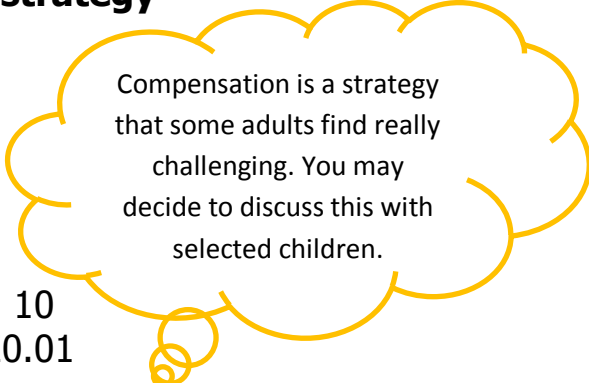
12.48 - 9.79

2,589 - 2,493

#### **Possible bridging strategy**

0.01 + 0.2 + 2.48

7 + 89



Compensation is a strategy that some adults find really challenging. You may decide to discuss this with selected children.

### Regrouping: compensation

#### **Example Calculation**

5,893,614 - 6,090

£280.00 - £4.99

#### **Possible strategy**

5,893,614 - 6,100 + 10

£280.00 - £5.00 + £0.01

### Regrouping: bridging through 60 to calculate a time interval

#### **Example Calculation**

Suzy wants to run one lap around the track as fast as her friend Amy, who can run one lap in 1.25 minutes. Suzy started running her lap around the track 47 seconds ago.

How much time does Suzy have left to finish her lap if she wants to tie Amy's time? (**See Year 5 number line example**)

### Vocabulary

Consolidate previous years

## Year 6

Children should choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

**Mental methods** should continue to be developed, supported by a concrete – pictorial – abstract approach. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency.

### Written methods

As year 5, progressing to larger numbers; aiming for both conceptual understanding and procedural fluency with a secure columnar method.

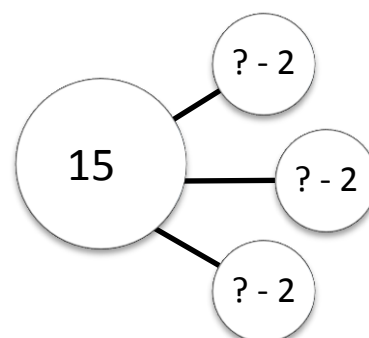
$$\begin{array}{r}
 56123.413 \\
 - 4814.05 \\
 \hline
 1418.38
 \end{array}$$

Continue calculating with decimals, including those with different numbers of decimal places.

### Missing number problems

$$10,000,000 = 9,000,100 + \square \quad (7 - 2) \times 3 = \square$$

$$(\square - 2) \times 3 = 15$$



1		9		7
-		3	4	5
2		8		

## Progression from Mental Methods to Written Methods for Multiplication

To multiply successfully, children need to be able to:

- recall all multiplication facts to  $12 \times 12$
- partition number into multiples of one hundred, ten and one
- 
- work out products such as  $70 \times 5$ ,  $70 \times 50$ ,  $700 \times 5$  or  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value

### Mental Skills

Recognise the size and position of numbers

Count on in different steps 2s, 5s, 10s

Double numbers up to 10

Recognise multiplication as repeated addition

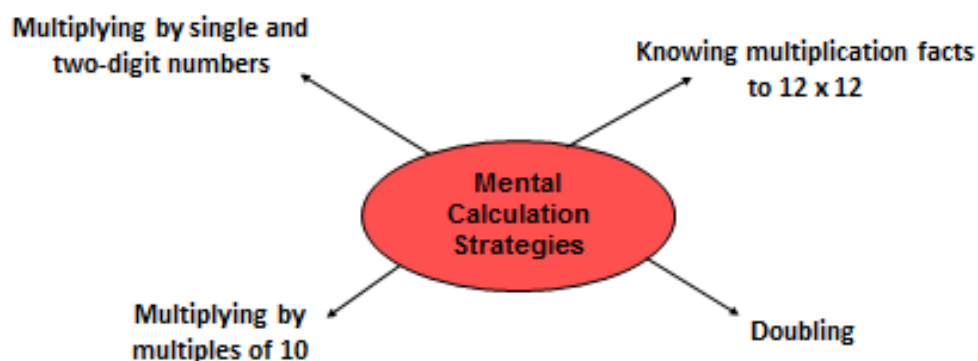
Quick recall of multiplication facts

Use known facts to derive associated division facts

Use known facts to generate other facts (e.g. double the 2 x table to find 4 x table)

Multiplying by 10, 100, 1000 and understanding the effect

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication. The children should decide the most efficient method to use, e.g. mental, jotting or written.



### Concrete resources and pictorial representations

Counting apparatus

Place value apparatus

Place value cards

Number tracks

Numbered number lines

Marked but unnumbered number lines

Empty number lines

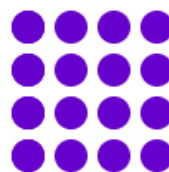
Hundred square

Counting stick

Bead string

100 square

Multiplication squares



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



# Multiplication Strategies

## **Features of multiplication**

There are three different ways of thinking about multiplication:

- as repeated addition, for example  $3 + 3 + 3 + 3$
- as an array, for example four rows of three objects
- as a scaling factor, for example, making a line 3 cm long four times as long.

The use of the multiplication sign can cause difficulties. Strictly,  $3 \times 4$  means four threes or  $3 + 3 + 3 + 3$ . Read correctly, it means 3 multiplied by 4.

However, colloquially it is read as '3 times 4', which is  $4 + 4 + 4$  or three fours.

Fortunately, multiplication is commutative:  $3 \times 4$  is equal to  $4 \times 3$ , so the outcome is the same. It is also a good idea to encourage children to think of any product either way round, as  $3 \times 4$  or as  $4 \times 3$ , as this reduces the facts that they need to remember by half.

A useful link between multiplication and addition allows children to work out new facts from facts that they already know. For example, the child who can work out the answer to  $8 \times 6$  (six eights) by recalling  $8 \times 5$  (five eights) and then adding 8 will, through regular use of this strategy, become more familiar with the fact that  $8 \times 6$  is 48.

Another feature of multiplication occurs in an expression such as  $(4 + 5) \times 3$ , which involves both multiplication and addition. The distributive law of multiplication over addition means that:

$$(4 + 5) \times 3 = (4 \times 3) + (5 \times 3)$$

This feature can be very useful in mental calculations.

## **Multiplication facts to 12 x 12**

Fluent recall of multiplication facts relies on regular opportunities for practice. Generally, frequent short sessions are more effective than longer, less frequent sessions. It is crucial that the practice involves as wide a variety of activities, situations, questions and language as possible and that it leads to deriving and recognising number properties, such as doubles and halves, odd and even numbers, multiples, factors and primes.

## **Doubling**

The ability to double numbers is useful for multiplication.

Historically, multiplication was carried out by a process of doubling and adding. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations.

Sometimes it can be helpful to halve one of the numbers in a multiplication calculation and double the other.

## Multiplying by multiples of 10

Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication facts. This ability is fundamental to being able to multiply larger numbers.

- Children are not told to add zeros. They understand scale factors.

## Multiplying by single-digit and by two-digit numbers

Once children are familiar with some multiplication facts, they can extend their skills.

- One strategy is to partition one of the numbers and use the **distributive law** of multiplication over addition. So, for example,  $6 \times 7 = 6 \times (5 + 2) = 6 \times 5 + 6 \times 2$  or, in words, 'seven sixes are five sixes plus two sixes'.
- Another strategy is to make use of factors, so  $7 \times 6$  is seen as  $7 \times 3 \times 2$ .

Once children understand the effect of multiplying by 10, they can start to extend their multiplication skills to larger numbers.

- A product such as  $26 \times 3$  can be worked out by regrouping 26 into  $20 + 6$ , multiplying each part by 3, then recombining.
- One strategy for multiplication by 2, 4, 8, 16, 32, ... is to use doubling, so that  $9 \times 8$  is seen as  $9 \times 2 \times 2 \times 2$ .
- A strategy for multiplying by 50 is to multiply by 100, then halve, and for multiplying by 25 is to multiply by 100 then divide by 4.

Since each of these strategies involves at least two steps, most children will find it helpful to make jottings of the intermediate steps in their calculations.

### Tracking recall of Multiplication Facts

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Teaching of Mathematics

x	2	3	4	5	6	7	8	9	11	12
2	4	6	8	10	12	14	16	18	22	24
3		9	12	15	18	21	24	27	33	36
4			16	20	24	28	32	36	44	48
5				25	30	35	40	45	55	60
6					36	42	48	54	66	72
7						49	56	63	77	84
8							64	72	88	96
9								81	99	108
11									121	132
12										144

19 facts to learn in Y2

21 facts to learn in Year 3

15 Facts to learn in Year 4

## Year 3

### **Multiplication facts to 12 x 12**

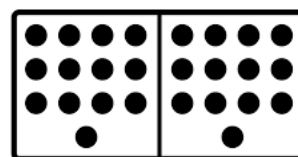
#### **Expectations**

- Derive and recall doubles of any two-digit number
- Derive and recall doubles of all multiples of 50 to 500
- Derive and recall multiplication facts for the 2, 10, 5, 3, 4 and 8 times-tables
- Recognise multiples of 2, 10, 5, 3, 4 and 8 up to the twelfth multiple

### **Doubling**

#### **Expectations**

- Double multiples of 10 to 100, e.g. double 90
- Double multiples of 5 to 100, e.g. double 35
- Double any two-digit number e.g. double 47
- Doubles all multiples of 50 to 500



### **Multiplying by multiples of 10**

#### **Expectations**

- Multiply one-digit and two-digit numbers by 10 or 100, e.g.  $7 \times 100$ ,  $46 \times 10$ ,  $54 \times 100$
- Change pounds to pence, e.g. £6 to 600 pence, £1.50 to 150 pence

### **Vocabulary**

Partition, grid method, inverse (consolidate previous years)

### **Developing understanding**

**Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

They should begin to use estimation to check answers to calculations and their growing knowledge of number properties e.g. whether the answer will be odd or even.

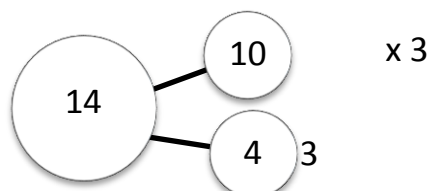
Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of  $\frac{1}{10}$ , 0.1 and £0.10.

They should practise times table facts and continue to notice patterns.

## Year 3

Practical resources should continue to be used to support thinking. The use of informal jottings and drawings to solve problems should be encouraged, including the bar model (see year 2). The children will continue to choose and use regrouping methods.

$14 \times 3$



$$\begin{array}{r} 25 \times 5 = \\ 10 \times 5 = 50 \\ 10 \times 5 = 50 \\ 5 \times 5 = +25 \\ \hline 125 \end{array}$$

The children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$7 \times 4 = \square$

$\square = 4 \times 7$

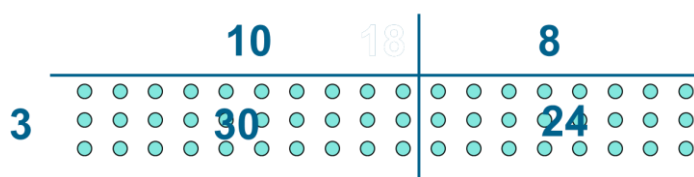
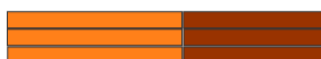
$\square \times 4 = 28$

$28 = 4 \times \square$

### Written methods (progressing to 2d x 1d)

Develop written methods using an understanding of visual images; squared paper and arrays

$18 \times 3$



$30 + 24 = 54$

When this understanding is in place progress to the grid method.

	10	8
3	30	24

Give the children plenty of opportunities to explore this and deepen their understanding by using Numicon Shapes, rods, Dienes apparatus and place value counters.

## Year 4

### **Multiplication facts to 12 x 12**

#### **Expectations**

- Derive and recall multiplication facts up to 12 x 12
- Recognise multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 up to the twelfth multiple
- Recognise and use factor pairs and commutativity in mental calculations
- Use place value, known and derived facts to multiply mentally, including:
  - multiplying by 0 and 1
  - multiplying together three numbers

### **Doubling**

#### **Expectations**

- Use regrouping to double any number, including decimals to one decimal place

### **Multiplying by multiples of 10**

#### **Expectations**

- Multiply numbers to 1000 by 10 then 100 e.g. 325 x 100
- Multiply a multiple of 10 to 100 by a single-digit number e.g. 60 x 3, 50 x 7
- Change hours to minutes; convert between units involving multiples of 10 and 100, e.g. centimetres and millimetres, centilitres and millilitres, and convert between pounds and pence, metres and centimetres, e.g. £5.99 to 599 pence, 2.5m to 250cm

### **Multiplying by single-digit and by two-digit numbers**

#### **Expectations**

- Multiply numbers to 20 by a single-digit number, e.g. 17 x 6

### **Vocabulary**

Factor (consolidate previous years)

### **Developing understanding**

**Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties e.g. whether the answer will be odd or even.

## Year 4

### Developing understanding

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100, 0.01 and £0.01.

They become fluent and confident to recall all tables up to 12 x 12.

They can multiply 3 small numbers together.

They should be encouraged to choose from a range of strategies:

- Regrouping using x 10, x 20 etc.
- Doubling to solve x 2, x 4, x 8
- Recall of times tables
- Use of commutativity of multiplication

**They continue to use concrete resources and informal jottings to support their thinking.**

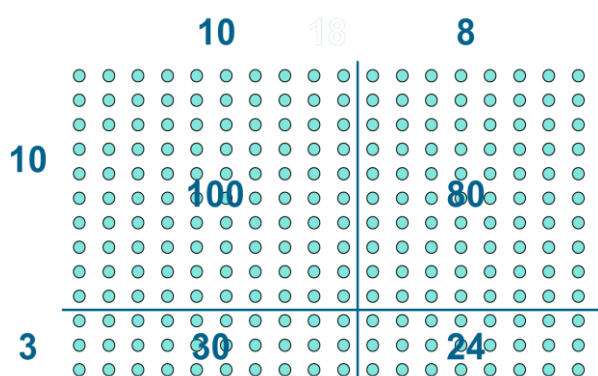
They should use their understanding of the inverse and concrete resources to solve missing number problems, e.g.

$$\square \times 5 = 160 \quad 32 \times \square = 160 \quad \square \square \times 8 = 160 \quad \square \square \times 4 = 160$$

They solve practical problems where they need to scale up using known number facts, e.g. how tall would a 25cm sunflower be if it grew 6 times taller? They might also choose to represent this with a bar model.

### Written methods (progressing to 3d x 2d)

Children continue to embed and deepen their understanding of the grid method to multiply 2d x 2d, **progressing to 3d x 2d**. Ensure this is still linked back to their understanding of arrays and area/squared paper



# Year 5

## **Multiplication facts to 12 x 12**

### **Expectations**

- Recall squares of numbers to 12 x 12
- Use multiplication facts to derive products of pairs of multiples of 10 and 100
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19

## **Doubling**

### **Expectations**

- Form equivalent calculations and use doubling and halving, e.g.
  - multiply by 4 by doubling twice, e.g.  $16 \times 4 = 32 \times 2 = 64$
  - multiply by 8 by doubling three times, e.g.  $12 \times 8 = 24 \times 4 = 48 \times 2 = 96$
  - multiply by 5 by multiplying by 10 then halving, e.g.  $18 \times 5 = 180 \div 2 = 90$
  - multiply by 20 by doubling then multiplying by 10, e.g.  $53 \times 20 = 106 \times 10 = 1060$
- Multiply by 50 by multiplying by 100 and halving
- Multiply by 25 by multiplying by 100 and halving twice
- Use Regrouping to double any number, including decimals to two decimal places

## **Multiplying by multiples of 10**

### **Expectations**

- Multiply whole numbers and decimals by 10, 100 or 1000, e.g.  $4.3 \times 10$ ,  $0.75 \times 100$
- Multiply pairs of multiples of 10, and a multiple of 100 by a single digit number, e.g.  $60 \times 30$ ,  $900 \times 8$
- Multiply by 25 or 50, e.g.  $48 \times 25$ ,  $32 \times 50$  using equivalent calculations, e.g.  $48 \times 100 \div 4$ ,  $32 \times 100 \div 2$
- Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6 kg to 2600 g, 3.5 cm to 35 mm, and 1.2 m to 120 cm

## **Multiplying by single-digit and by two-digit numbers**

### **Expectations**

- Multiply two-digit numbers by 4 or 8, e.g.  $26 \times 4$
- Multiply two-digit numbers by 5 or 20, e.g.  $32 \times 5$ ,  $14 \times 20$
- Multiply by 25 or 50, e.g.  $48 \times 25$ ,  $32 \times 50$

## **Vocabulary**

Cube numbers, prime numbers, square numbers, common factors, prime factors, composite (non-prime) numbers.

## Year 5

**Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).**

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties e.g. whether the answer will be odd or even.

### **Developing understanding**

Children should continue to count regularly, on and back, including steps of powers of 10.

They should be encouraged to choose from a range of strategies to solve problems mentally:

- Regrouping using  $\times 10$ ,  $\times 20$  etc.
- Doubling to solve  $\times 2$ ,  $\times 4$ ,  $\times 8$
- Recall of times tables
- Use of commutativity of multiplication

**They continue to use informal jottings to their support thinking.**

If children know the times table facts to  $12 \times 12$ . Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table).

Do they fully understand the effect of multiplying by 10, 100, 1000, including decimals?

Can they use practical resources and jottings to explore equivalent statements, e.g.  $4 \times 35 = 2 \times 2 \times 35$

They should identify factor pairs for numbers, recall prime numbers up to 19 and identify prime numbers up to 100 (with reasoning).

The children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$$\square \times 2 \times 50 = 1600 \quad 32 \times \square \square = 1600 \quad \square \square \times 8 \square = 1600 \quad \square \square \square \times 4 = 1600$$

They should also solve practical problems where they need to scale up by relating to known facts.

# Year 5

## Written methods (progressing to 4d x 2d)

Children explore how the grid method supports an understanding of short multiplication initially before progressing to long multiplication (for 2d x 2d) as shown below

x	100	20	3
7	700	140	21

		1	2	3	
	x			7	
		8	6	1	
		1	2		

The 'carry' may be crossed through if this helps the child to ensure it is used.

Progress to long multiplication and decimals

	10	8
10	100	80
3	30	24

		1	8	
	x	1	3	
		5 <sub>2</sub>	4	
	1	8	0	
	2	3	4	

	2	9	.	8				8	5	3	7		
x				6			x				2	9	
1	7	8	.	8			7	6 <sub>4</sub>	8 <sub>3</sub>	3 <sub>6</sub>	3		
1	5	4					1	7 <sub>1</sub>	0	7 <sub>1</sub>	4	0	
							2	4	7	5	7	3	
							1		1				

Again the 'carry' may be crossed through if this helps the child to ensure it is used. Some children may also benefit by leaving a blank row between each calculation line.

## Year 6

### **Multiplication facts to 12 x 12**

#### **Expectations**

- Recall squares of numbers to 12 x 12 and derive corresponding squares of multiples of 10
- Use place value and multiplication facts to derive related multiplication facts involving decimals e.g.  $0.8 \times 7$ ,  $0.006 \times 12$
- Identify common factors, common multiples and prime numbers.

### **Doubling**

#### **Expectations**

- Use regrouping to double any number, including decimals up to two decimal places.

### **Multiplying by multiples of 10**

#### **Expectations**

- Multiply pairs of multiples of 10 and 100, e.g.  $50 \times 30$ ,  $600 \times 20$
- Convert between units of measurement using decimals to two places, e.g. change 2.75 l to 2750 ml, or vice versa.

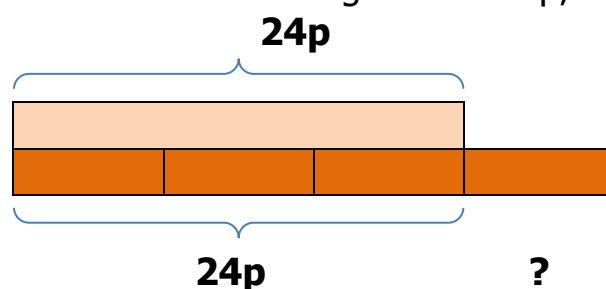
### **Multiplying by single-digit and by two-digit numbers**

#### **Expectations**

- Multiply a two-digit and a single-digit number, e.g.  $28 \times 7$
- Find new facts from given facts, e.g. given that three oranges cost 24p, find the cost of four oranges.

### **Vocabulary**

Consolidate previous years.



### **Developing understanding**

**Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method).**

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties e.g. whether the answer will be odd or even.

## Year 6

### Developing understanding

Children should experiment with the order of operations, investigating the effect of positioning the brackets in different places, e.g.  $20 - 5 \times 3 = 5$ ;  
 $(20 - 5) \times 3 = 45$

They should be encouraged to choose from a range of strategies to solve problems mentally:

- Regrouping using  $\times 10$ ,  $\times 20$  etc.
- Doubling to solve  $\times 2$ ,  $\times 4$ ,  $\times 8$
- Recall of times tables.
- Use of commutativity of multiplication.

They should be able to recite other times tables using known facts, e.g. the 13 times tables or the 24 times table.

The children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$$12.3 \times \square = 86.1 \quad 12. \square \times 3 = 86.1 \quad 12.3 \times 3 = 8\square. \square$$

They should identify common factors and multiples of given numbers.

The children should solve practical problems where they need to scale up by relating to known facts.

### Written methods

Continue to refine and deepen understanding of written methods including fluency for using short and long multiplication with increasingly larger numbers and decimals.

	2	9	.	8					8	5	3	7	
x				6			x				2	9	
1	7	8	.	8				7	6 <sub>4</sub>	8 <sub>3</sub>	3 <sub>6</sub>	3	
1	5	4					1	7 <sub>1</sub>	0	7 <sub>1</sub>	4	0	
							2	4	7	5	7	3	
							1		1				

See notes on Year 5 examples for variations in the written recording and also grid method for those children who may need an alternative approach.

## Progression from Mental Methods to Written Methods for Division

**To carry out written methods of division successful, children need to be able to:**

- understand division as repeated subtraction
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92
- multiply a two-digit number by a single-digit number mentally
- subtract numbers using the column method

### Mental Skills

Recognise the size and position of numbers

Count back in different steps 2s, 5s, 10s

Halve numbers to 20

Recognise division as repeated subtraction

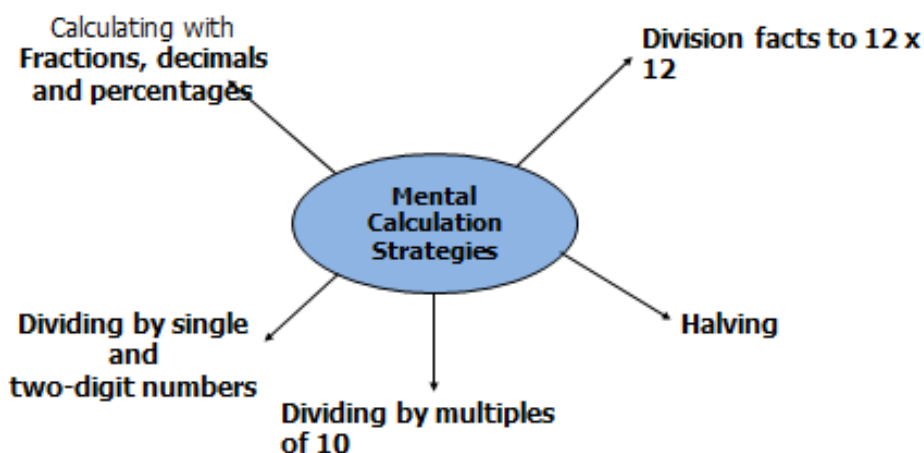
Quick recall of division facts

Use known facts to derive associated facts

Divide by 10, 100, 1000 and understanding the effect

Divide by multiples of 10

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.



### Concrete resources and pictorial representations

Counting apparatus

Place value apparatus

Arrays

100 squares

Number tracks

Numbered number lines

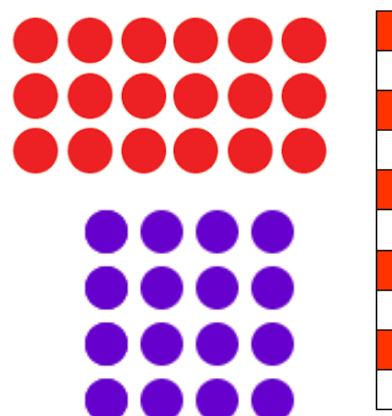
Marked but unnumbered lines

Empty number lines

Multiplication squares

Bead strings

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



# Division Strategies

## Features of division

There are three different ways of thinking about division:

- as equal **sharing**, for example regrouping into equal sets determined by the divisor.  $12 \div 4$  is seen as share 12 between 4
- as equal **grouping** or subtraction of the divisor, for example repeated subtraction of a group.  $12 \div 4$  is seen as how many groups of 4 are there in 12. It is also called the inverse of multiplication structure
- as **ratio** or a comparison of the scale of two quantities or measurements in which the quotient is regarded as a scale factor. This is the inverse of the scaling structure of multiplication, for example,  $12 \div 4$  is interpreted as how many times more (or less) is 12 than 4

## Division facts to 12 x 12

Fluent recall of division facts relies on regular opportunities for practice. Generally, frequent short sessions are more effective than longer, less frequent sessions. It is crucial that the practice involves as wide a variety of activities, situations, questions and language as possible and that it leads to deriving and recognising number properties, such as halves, odd and even numbers, multiples, factors and primes.

## Halving

Most people find doubles the easiest multiplication facts to remember, and therefore halving or regrouping into two groups is a way of securing an early understanding of division. It can later be developed as a structure to find other fractions of quantities.

## Dividing by multiples of 10

Being able to divide by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to divide larger numbers.

### **Dividing by single-digit numbers**

Once children understand the effect of multiplying and dividing by 10, they can start to extend their multiplication and division skills to larger numbers.

- One strategy for multiplication by 2, 4, 8, 16, 32, ... is to use doubling, so that  $9 \times 8$  is seen as  $9 \times 2 \times 2 \times 2$ . A strategy for dividing by the same numbers is to use halving.
- A strategy for dividing by 50 is to divide by 100, then double, and for dividing by 25 is to divide by 100 then double and double again.

Since each of these strategies involves at least two steps, most children will find it helpful to make jottings of the intermediate steps in their calculations.

### **Fractions, decimals and percentages**

Children need an understanding of how fractions, decimals and percentages relate to each other. For example, if they know that  $\frac{1}{2}$ , 0.5 and 50% are all ways of representing the same part of a whole, then they can see that the calculations;

half of 40

$\frac{1}{2} \times 40$

$40 \times \frac{1}{2}$

$40 \times 0.5$

$0.5 \times 40$

50% of 40

are different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.

There are strong links between this strategy and 'dividing by multiples of 10'.

Finding one-half of 12, for example 'Joe spends half of £12. How much does he spend?'



**Halving**

Joe spends £6

# Year 3

## **Division facts to 12 x 12**

### **Expectations**

- Derive and recall halves of any two-digit number
- Derive and recall halves of all multiples of 50 to 500
- Derive and recall division facts for the 2, 10, 5, 3, 4 and 8 times-tables
- Recognise multiples of 2, 10, 5, 3, 4 and 8 up to the twelfth multiple

## **Halving**

### **Expectations**

- Halve multiples of 10 to 100, e.g. halve 90
- Halve multiples of 5 to 100, e.g. halve 70
- Halve any two-digit number e.g. halve 47
- Halve all multiples of 50 to 500

## **Dividing by multiples of 10**

### **Expectations**

- Divide one-digit, two-digit and three-digit numbers by 10 and 100, e.g.  $70 \div 10$ ,  $260 \div 10$ ,  $750 \div 100$
- Change pence to pounds, e.g. 600 pence to £6, 150 pence to £1.50

## **Fractions, decimals and percentages**

### **Expectations**

- Find half of all numbers to 100 e.g. half of 54
- Find a  $\frac{1}{4}$ ,  $\frac{3}{4}$ ,  $\frac{1}{10}$ ,  $\frac{1}{5}$ ,  $\frac{1}{3}$  and  $\frac{1}{8}$  of numbers in the 2, 3, 4, 5, 8 and 10 times tables up to the twelfth multiple

## **Vocabulary**

Inverse, *consolidate language from Y1 and Y2*

## **Developing understanding**

**Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties

Children should count regularly, on and back, in steps of 3, 4 and 8.

They are encouraged to use what they know about known times table facts to work out other times tables. This then helps them to make new connections (e.g. through **halving** they make connections between the 2, 4 and 8 times tables).

Children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$$30 \times \square = 60$$

$$\square \div 3 = 20$$

$$\square = 60 \div 30$$

$$60 = \square \div \nabla$$

# Year 3

## Developing understanding

They should be given opportunities to solve **grouping** and **sharing** problems practically, including where there is a remainder but the answer needs to be given as a whole number (adjusted) e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.

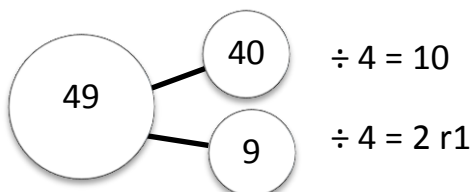
## Towards written methods

How many 6s are there in 30 can be modelled as:

Children need to be able to regroup the dividend in different ways.

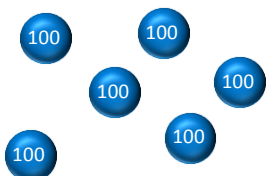
$$48 \div 4 = 12$$

$$49 \div 4 = 12 \text{ r}1$$



Place value counters can be used to support children apply their knowledge of grouping and sharing (like Numicon Shapes the counters are a grouped resource):

$600 \div 100 =$  How many groups of 100 in 600?



They may also use jottings to model the physical process

$$\begin{array}{r} 216 \\ \swarrow \quad \searrow \\ 200 \quad 16 \\ \div 2 \quad \downarrow \quad \div 2 \\ 100 \quad + \quad 8 = 108 \end{array}$$

## Formal written methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number. As with all standard methods the numbers chosen should reflect the progression e.g. use numbers that do not require an exchange or 'carry' until the understanding is secure.

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends.

Place value counters for 1312: three blue '100' counters, three red '10' counters, and two green '1' counters.

$$\begin{array}{r} 437 \text{ r}1 \\ 3 \overline{) 1312} \\ \underline{12} \phantom{00} \\ 11 \phantom{00} \\ \underline{9} \phantom{00} \\ 21 \phantom{00} \\ \underline{21} \phantom{00} \\ 2 \end{array}$$

# Year 4

## **Division facts to 12 x 12**

### **Expectations**

- Derive and recall division facts up to 12 x 12
- Recognise multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 up to the twelfth multiple
- Use place value, known and derived facts to multiply mentally

## **Halving**

### **Expectations**

- Use regrouping to halve any number, including decimals to one decimal place
- Find halves of multiples of 10 and 100, e.g. half of 1600, half of 680

## **Dividing by multiples of 10**

### **Expectations**

- Divide numbers to 1000 by 10 then 100 (whole number answers), e.g.  $120 \div 10$ ,  $600 \div 100$ ,  $850 \div 10$
- Change minutes to hours; convert between units involving multiples of 10 and 100, e.g. millimetres and centimetres, millilitres and centilitres, and convert between pence and pounds, centimetres and metres, e.g. 599 pence to £5.99, 250cm to 2.5m

## **Dividing two-digit numbers by a single-digit number**

### **Expectations**

- Use knowledge of relationships between 2, 4 and 8 to find  $\frac{1}{2}$ ,  $\frac{1}{4}$  and  $\frac{1}{8}$  of three-digit numbers

## **Fractions, decimals and percentages**

### **Expectations**

- Find half of all even numbers to 1000 e.g. half of 654
- Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g.  $\frac{3}{8}$  of 24
- Recall fraction and decimal equivalents for one-half, quarters, tenths and hundredths, e.g. recall the equivalence of 0.3 and  $\frac{3}{10}$ , and 0.03 and  $\frac{3}{100}$

## **Vocabulary**

Divide, divided by, divisible by, divided into, share between, groups of, factor, factor pair, multiple, times as (big, long, wide ...etc.), equals, remainder, quotient, divisor, inverse

# Year 4

## Developing understanding

Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties e.g. whether the answer will be odd or even.

Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 11, 12, 25 and 1000.

They should know the division facts for the times-tables to 12 x 12

Children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$$30 \times \square\square = 600$$

$$\square\square\square \div 3 = 200$$

$$\square\square = 600 \div 30$$

$$600 = \square\square \div \nabla\nabla$$

## Towards written methods

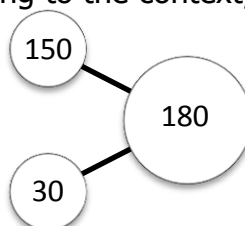
Children will continue to explore division as **sharing** and **grouping**, and to represent calculations on a number line until they have a secure understanding. They should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
  1. Dividend just over 10 lots of the divisor, e.g.  $84 \div 7$
  2. Dividend just over 10 lots of the divisor when the divisor is a teen number, e.g.  $180 \div 15$

All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context, i.e. rounded up or down to relate to the answer to the problem

$$180 \div 15 = 12$$



*Jottings*

$$15 \times 2 = 30$$

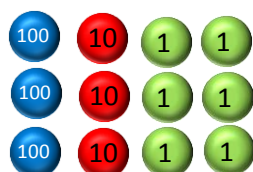
$$15 \times 5 = 75$$

$$15 \times 10 = 150$$

## Formal written methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number. As with all standard methods the numbers chosen should reflect the progression e.g. use numbers that do not require an exchange or 'carry' until the understanding is secure.

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends.



$$\begin{array}{r} 151 \\ 3 \overline{) 453} \\ \underline{3 \phantom{00}} \\ 15 \phantom{0} \\ \underline{15 \phantom{0}} \\ 3 \end{array}$$

$$453 \div 3 =$$



exchange 1 blue into 10 green

$$\begin{array}{r} 151 \\ 3 \overline{) 453} \\ \underline{15} \phantom{0} \\ 15 \phantom{0} \\ \underline{15} \phantom{0} \\ 3 \end{array}$$

# Year 5

## Division facts to 12 x 12

### **Expectations**

- Use known multiplication facts to derive corresponding division facts from pairs of multiples of 10 and 100

## Halving

### **Expectations**

- Form equivalent calculations and use halving, e.g.
  - divide by 4 by halving twice, e.g.  $104 \div 4 = 52 \div 2 = 26$
  - divide by 8 by halving three times, e.g.  $104 \div 8 = 52 \div 4 = 26 \div 2 = 13$
  - divide by 5 by dividing by 10 then doubling, e.g.  $185 \div 10 = 18.5 \times 2 = 37$
- Dividing by 50 by dividing by 100 and doubling
- Dividing by 25 by dividing by 100 and doubling twice

## Dividing by multiples of 10

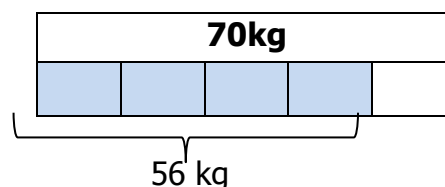
### **Expectations**

- Divide whole numbers and decimals by 10, 100 or 1000, e.g.  $85 \div 10$ ,  $673 \div 100$
- Divide a multiple of 10 by a single-digit number (whole number answers) e.g.  $80 \div 4$ ,  $270 \div 3$
- Convert smaller units of measurement to larger units using decimals to two places, e.g. change 2600 g to 2.6 kg, 35 mm to 3.5 cm, and 125 cm to 1.25 m

## Dividing two-digit numbers by a single-digit number

### **Expectations**

- Divide two-digit numbers by 4 or 8, e.g.  $96 \div 84$



## Fractions, decimals and percentages

### **Expectations**

- Find non-unit fractions or whole numbers or quantities, e.g.  $\frac{4}{5}$  of 70 kg
- Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of £80

## Vocabulary

Common factors, prime number, prime factors, composite numbers, short division, square number, cube number, inverse, power of

**Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)**

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties e.g. whether the answer will be odd or even.

# Year 5

## Developing understanding

Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency.

They should learn and apply the multiplication facts to 12 x 12 and corresponding division facts.

Children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$$30 \times \square\square\square = 645 \quad \square\square\square \div 9 = 200 \quad \square\square\square = 645 \div 30 \quad 324 = \square\square\square \div \nabla\nabla$$

## Towards written methods

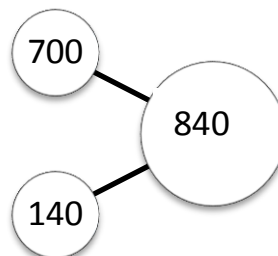
Children will continue to explore division as **sharing** and **grouping**, and to represent calculations with a jotting until they have a secure understanding. They should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
  1. Dividend just over 10 lots of the divisor, e.g.  $84 \div 7$
  2. Dividend just over 10 lots of the divisor when the divisor is a teen number, e.g.  $173 \div 15$
  3. Dividend over 100 lots of the divisor, e.g.  $840 \div 7$
  4. Dividend over 20 lots of the divisor, e.g.  $168 \div 7$

All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context, i.e. rounded up or down to relate to the answer to the problem

$$840 \div 7 = 120$$



*Jottings*

$$7 \times 100 = 700$$

$$7 \times 10 = 70$$

$$7 \times 20 = 140$$

## Formal Written Methods

These continue to be developed with an understanding of place value and the language of sharing and grouping.

$453 \div 3 =$

exchange 1 blue into 10 green

$$\begin{array}{r} 151 \\ 3 \overline{) 453} \end{array}$$

$$1435 \div 6$$

		2	3	9	r	1
6	1	4	<sup>2</sup> 3	<sup>5</sup> 5		

Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this, e.g. what could I do with this remaining 1? How could I share this between 6 as well?

# Year 6

## **Division facts to 12 x 12**

### **Expectations**

- Use place value and multiplication facts to derive related division facts involving decimals e.g.  $4.8 \div 6$

## **Halving**

### **Expectations**

- Use regrouping to halve any number, including decimals up to two decimal places
- Form equivalent calculations and use doubling and halving, e.g.
  - divide by 25 by dividing by 100 then multiplying by 4 e.g.  
 $460 \div 25 = 4.6 \times 4 = 18.4$
  - divide by 50 by dividing by 100 then doubling e.g.  
 $270 \div 50 = 2.7 \times 2 = 5.4$

## **Dividing by multiples of 10**

### **Expectations**

- Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g.  $600 \div 20$ ,  $800 \div 400$ ,  $2100 \div 300$
- Divide by 25 or 50
- Convert between units of measurement using decimals to two places, e.g. change 2750 ml to 2.75 l

## **Dividing two-digit numbers by a single-digit number**

### **Expectations**

- Divide a two-digit number by a single-digit number e.g.  $68 \div 4$
- Find new facts from given facts, e.g. given that three oranges cost 24p, find the cost of four oranges

## **Fractions, decimals and percentages**

### **Expectations**

- Recall equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or  $35/100$
- Find half of decimals with units and tenths, e.g. half of 3.2
- Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g

## **Vocabulary**

Consolidate previous years

# Year 6

## Developing understanding

Children should be encouraged to choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)

They should use estimation and inverse operations to check answers to calculations alongside their growing knowledge of number properties e.g. whether the answer will be odd or even.

Children should continue to count regularly rehearsing all the steps covered in Y3 – Y5. They should apply the multiplication facts to 12 x 12 and corresponding division facts.

The children should use their understanding of the inverse and practical resources to solve missing number problems, e.g.

$$7 \times \square\square\square = 1,736 \quad \square\square\square\square \div 9 = 12,789 \quad 6324 = \square\square\square\square \div \nabla\nabla$$

They should use their knowledge of the **rules of divisibility**

## Towards written methods

The children will continue to explore division as **sharing** and **grouping**, and to represent calculations using jottings when appropriate. Quotients should be expressed as decimals and fractions.

## Formal Written Methods

These continue to be developed with an understanding of place value and the language of sharing and grouping. **If necessary children will still support their thinking with place value counters until they are secure with the methods.** They will express the remainder as a decimal or a fraction.

- **Short division** is used when the divisor is a known 'table' i.e. 2 - 12

<p>432 ÷ 5 becomes</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{40} \phantom{2} \\ 32 \\ \underline{30} \\ 2 \end{array}$ <p>Answer: 86 remainder 2</p>	<p>496 ÷ 11 becomes</p> $\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \\ \underline{44} \phantom{6} \\ 56 \\ \underline{55} \\ 1 \end{array}$ <p>Answer: 45 <math>\frac{1}{11}</math></p>
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- **Long division** The 'arrow' method builds on a secure understanding of short division. It ignores the value of the digit. It is the preferred method for the end of Year 6. However, if children have difficulty progressing to this method you may choose to use the long division method that links to chunking.

<p style="text-align: center;">432 ÷ 15 becomes</p> <p><b>Arrow method</b></p> $\begin{array}{r} 28 \cdot 8 \\ 15 \overline{) 4320} \\ \underline{30} \phantom{0} \downarrow \\ 132 \\ \underline{150} \phantom{0} \downarrow \\ 120 \\ \underline{120} \\ 0 \end{array}$ <p style="text-align: center;">Answer: 28.8</p>	<p style="text-align: center;">432 ÷ 15 becomes</p> <p><b>'chunking'</b></p> $\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30} \phantom{0} \phantom{0} 15 \times 20 \\ \underline{132} \phantom{0} \\ 120 \phantom{0} 15 \times 8 \\ \underline{120} \\ 0 \end{array}$ <p style="text-align: center;">Answer: 28 <math>\frac{4}{5}</math></p>
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# Glossary of Terms

(taken from *Mathematics glossary for teachers in Key Stages 1 to 3* published by NCETM January 2014)

<b>addend</b> (KS1)	A number to be added to another. See also <b>dividend</b> , <b>subtrahend</b> and <b>multiplicand</b> .																																									
<b>addition</b> (KS1)	<p>The binary operation of addition on the set of all real numbers that adds one number of the set to another in the set to form a third number which is also in the set. The result of the addition is called the <b>sum</b> or <b>total</b>. The operation is denoted by the + sign. When we write 5 + 3 we mean 'add 3 to 5'; we can also read this as '5 plus 3'. In practice the order of addition does not matter: The answer to 5 + 3 is the same as 3 + 5 and in both cases the sum is 8. This holds for all pairs of numbers and therefore the operation of addition is said to be <b>commutative</b>.</p> <p>To add three numbers together, first two of the numbers must be added and then the third is added to this intermediate sum. For example, (5 + 3) + 4 means 'add 3 to 5 and then add 4 to the result' to give an overall total of 12. Note that 5 + (3 + 4) means 'add the result of adding 4 to 3 to 5' and that the total is again 12. The brackets indicate a priority of sub-calculation, and it is always true that (a + b) + c gives the same result as a + (b + c) for any three numbers a, b and c. This is the <b>associative</b> property of addition. Addition is the <b>inverse</b> operation to subtraction, and vice versa.</p> <p>There are two models for addition: <b>Augmentation</b> is when one quantity or measure is increased by another quantity. i.e. "I had £3.50 and I was given £1, then I had £4.50".</p> <p><b>Aggregation</b> is the combining of two quantities or measures to find the total. E.g. "I had £3.50 and my friend had £1, we had £4.50 altogether."</p>																																									
<b>array</b> (KS1)	An ordered collection of counters, numbers etc. in rows and columns.																																									
<b>associative</b> (KS1)	<p>A binary operation * on a set S is associative if a * (b * c) = (a * b) * c for all a, b and c in the set S. Addition of real numbers is associative which means a + (b + c) = (a + b) + c for all real numbers a, b, c. It follows that, for example, 1 + (2 + 3) = (1 + 2) + 3.</p> <p>Similarly multiplication is associative.</p> <p>Subtraction and division are not associative because: 1 - (2 - 3) = 1 - (-1) = 2, whereas (1 - 2) - 3 = (-1) - 3 = -4 and 1 ÷ (2 ÷ 3) = 1 ÷ 2/3 = 3/2, whereas (1 ÷ 2) ÷ 3 = (1/2) ÷ 3 = 1/6.</p>																																									
<b>binary operation</b> (KS1)	A rule for combining two numbers in the set to produce a third also in the set. Addition, subtraction, multiplication and division of real numbers are all binary operations.																																									
<b>brackets</b> (KS2)	<p>Symbols used to group numbers in arithmetic or letters and numbers in algebra and indicating certain operations as having priority.</p> <p>Example: 2 x (3 + 4) = 2 x 7 = 14 whereas 2 x 3 + 4 = 6 + 4 = 10.</p> <p>Example: 3(x + 4) denotes the result of adding 4 to a number and then multiplying by 3; (x + 1)2 denotes the result of adding 1 to a number and then squaring the result</p>																																									
<b>columnar addition or subtraction</b> (KS2)	<p>A <b>formal method</b> of setting out an addition or a subtraction in ordered columns with each column representing a decimal place value and ordered from right to left in increasing powers of 10.</p> <p>With addition, more than two numbers can be added together using column addition, but this extension does not work for subtraction.</p> <div><div><p>789 + 642 becomes</p><table><tr><td>7</td><td>8</td><td>9</td></tr><tr><td>+</td><td>6</td><td>4</td><td>2</td></tr><tr><td colspan="4"><hr/></td></tr><tr><td>1</td><td>4</td><td>3</td><td>1</td></tr><tr><td colspan="4"><hr/></td></tr><tr><td>1</td><td>1</td><td colspan="2"></td></tr></table><p>Answer: 1431</p></div><div><p>932 - 457 becomes</p><table><tr><td>8</td><td>12</td><td>1</td></tr><tr><td>9</td><td>3</td><td>2</td></tr><tr><td>-</td><td>4</td><td>5</td><td>7</td></tr><tr><td colspan="4"><hr/></td></tr><tr><td>4</td><td>7</td><td>5</td><td></td></tr></table><p>Answer: 475</p></div></div> <p>(Examples taken from Appendix 1 of the Primary National Curriculum for Mathematics)</p>	7	8	9	+	6	4	2	<hr/>				1	4	3	1	<hr/>				1	1			8	12	1	9	3	2	-	4	5	7	<hr/>				4	7	5	
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<b>common factor</b> (KS2)	<p>A number which is a factor of two or more other numbers, for example 3 is a common factor of the numbers 9 and 30</p> <p>This can be generalised for algebraic expressions: for example (x - 1) is a common factor of (x - 1)2 and (x - 1)(x + 3).</p>																																									
<b>common multiple</b> (KS2)	An integer which is a multiple of a given set of integers, e.g. 24 is a common multiple of 2, 3, 4, 6, 8 and 12.																																									

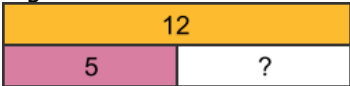
<b>commutative</b> (KS1)	A binary operation $*$ on a set $S$ is commutative if $a * b = b * a$ for all $a$ and $b \in S$ . Addition and multiplication of real numbers are commutative where $a + b = b + a$ and $a \times b = b \times a$ for all real numbers $a$ and $b$ . It follows that, for example, $2 + 3 = 3 + 2$ and $2 \times 3 = 3 \times 2$ . Subtraction and division are not commutative since, as counter examples, $2 - 3 \neq 3 - 2$ and $2 \div 3 \neq 3 \div 2$ .
<b>compensation (in calculation)</b> (KS1/2)	A mental or written calculation strategy where one number is rounded to make the calculation easier. The calculation is then adjusted by an appropriate compensatory addition or subtraction. Examples: <ul style="list-style-type: none"> <li>• <math>56 + 38</math> is treated as <math>56 + 40</math> and then 2 is subtracted to compensate.</li> <li>• <math>27 \times 19</math> is treated as <math>27 \times 20</math> and then 27 (i.e. <math>27 \times 1</math>) is subtracted to compensate.</li> <li>• <math>67 - 39</math> is treated as <math>67 - 40</math> and then 1 is added to compensate.</li> </ul>
<b>complement (in addition)</b> (KS2)	In addition, a number and its complement have a given total. Example: When considering complements in 100, 67 has the complement 33, since $67 + 33 = 100$
<b>concrete objects</b> (KS1)	Objects that can be handled and manipulated to support understanding of the structure of a mathematical concept. Materials such as Dienes (Base 10 materials), Cuisenaire, Numicon, pattern blocks are all examples of concrete objects.
<b>correspondence problems</b> (KS2)	Correspondence problems are those in which $m$ objects are connected to $n$ objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
<b>cube</b> (KS1/2)	In number and algebra, the result of multiplying to power of three, $n^3$ is read as 'n cubed' or 'n to the power of three' Example: Written 23, the cube of 2 is $(2 \times 2 \times 2) = 8$ .
<b>cube number</b> (KS2)	A number that can be expressed as the product of three equal integers. Example: $27 = 3 \times 3 \times 3$ . Consequently, 27 is a cube number; It is the cube of 3 or 3 cubed. This is written compactly as $27 = 3^3$ , using index, or power, notation.
<b>cube root</b> (KS3)	A value or quantity whose cube is equal to a given quantity. Example: the cube root of 8 is 2 since $2^3 = 8$ . This is recorded as $\sqrt[3]{8} = 2$ or $8^{1/3} = 2$
<b>decomposition</b> (KS2)	See <b>subtraction</b> by decomposition
<b>degree of accuracy</b> (KS2)	A measure of the precision of a calculation, or the representation of a quantity. A number may be recorded as accurate to a given number of decimal places, or rounded to the nearest integer, or to so many significant figures.
<b>difference</b> (KS1)	In mathematics (as distinct from its everyday meaning), difference means the numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another. E.g. the difference between 12 and 5 is 7; 12 is 5 more than 7 or 7 is 5 fewer than 12. Difference is one way of thinking about subtraction and can, in some circumstances, be a more helpful image for subtraction than 'take-away' e.g. $102 - 98$
<b>distributive</b> (KS2)	One binary operation $*$ on a set $S$ is distributive over another binary operation $\bullet$ on that set if $a * (b \bullet c) = (a * b) \bullet (a * c)$ for all $a, b$ and $c \in S$ . For the set of real numbers, multiplication is distributive over addition and subtraction since $a(b + c) = ab + ac$ for all $a, b$ and $c$ real numbers. It follows that $4(50 + 6) = (4 \times 50) + (4 \times 6)$ and $4 \times (50 - 2) = (4 \times 50) - (4 \times 2)$ . For division $\frac{(a + b)}{c} = \frac{a}{c} + \frac{b}{c} \text{ (division is distributive over addition)}$ <p>But</p> $\frac{c}{(a+b)} \neq \frac{c}{a} + \frac{c}{b} \text{ (addition is not distributive over division)}$ <p>Addition, subtraction and division are not distributive over other number operations.</p>
<b>divide</b> (KS1)	To carry out the operation of division.
<b>dividend</b> (KS1)	In division, the number that is divided. E.g. in $15 \div 3$ , 15 is the dividend See also Addend, subtrahend and multiplicand.

<b>divisibility</b> (KS2)	The property of being divisible by a given number. Example: A test of divisibility by 9 checks if a number can be divided by 9 with no remainder.
<b>divisible (by)</b> (KS2)	A whole number is divisible by another if there is no remainder after division and the result is a whole number. Example: 63 is divisible by 7 because $63 \div 7 = 9$ remainder 0. However, 63 is not divisible by 8 because $63 \div 8 = 7.875$ or 7 remainder 7.
<b>division</b> (KS1)	1. An operation on numbers interpreted in a number of ways. Division can be sharing – the number to be divided is shared equally into the stated number of parts; or grouping – the number of groups of a given size is found. Division is the inverse operation to multiplication. 2. On a scale, one part. Example: Each division on a ruler might represent a millimetre.
<b>divisor</b> (KS2)	The number by which another is divided. Example: In the calculation $30 \div 6 = 5$ , the divisor is 6. In this example, 30 is the dividend and 5 is the quotient.
<b>double</b> (KS1)	1. To multiply by 2. Example: Double 13 is $(13 \times 2) = 26$ . 2. The number or quantity that is twice another. Example: 26 is double 13. In this context, a 'near double' is one away from a double. Example: 27 is a near double of 13 and of 14. (N.B. spotting near doubles can be a useful mental calculation strategy e.g. seeing $25 + 27$ as 2 more than double 25.
<b>efficient methods</b> (KS2)	A means of calculation (which can be <b>mental or written</b> ) that achieves a correct answer with as few steps as possible. In written calculations, this often involves setting out calculations in a columnar layout. If a calculator is used the most efficient method uses as few key entries as possible.
<b>equal</b> (KS1)	Symbol: =, read as 'is equal to' or 'equals'. and meaning 'having the same value as'. Example: $7 - 2 = 4 + 1$ since both expressions, $7 - 2$ and $4 + 1$ have the same value, 5.
<b>equation</b> (KS3)	A mathematical statement showing that two expressions are equal. The expressions are linked with the symbol =. Examples: $7 - 2 = 4 + 1$ $4x = 3x^2 - 2x + 1 = 0$
<b>error</b> (KS3)	1. The difference between an accurate calculation and an approximate calculation or estimate; the difference between an exact representation of a number and an approximation to it obtained by rounding or some other process. In a calculation, if all numbers are rounded to some degree of accuracy the errors become more significant. 2. A mistake
<b>estimate</b> (KS2)	1. Verb: To arrive at a rough or approximate answer by calculating with suitable approximations for terms or, in measurement, by using previous experience. 2. Noun: A rough or approximate answer.
<b>even number</b> (KS1)	An integer that is divisible by 2.
<b>exchange</b> (KS2)	Change a number or expression for another of equal value. The process of exchange is used in some standard compact methods of calculation. Examples: 'carrying figures' in addition, multiplication or division; and ' <b>decomposition</b> ' in subtraction.
<b>exponent</b> (KS3)	Also known as index, a number, positioned above and to the right of another (the base), indicating repeated multiplication when the exponent is a positive integer. Example 1: $n^2$ indicates $n \times n$ ; and 'n to the (power) 4', that is $n^4$ means $n \times n \times n \times n$ . Example 2: since $2^5 = 32$ we can also think of this as '32 is the fifth power of 2'. Any positive number to power 1 is the number itself; $x^1 = x$ , for any positive value of x. Exponents may be negative, zero, or fractional. Negative integer exponents are the reciprocal of the corresponding positive integer exponent, for example, $2^{-1} = \frac{1}{2}$ . Any positive number to power zero equals 1; $x^0 = 1$ , for any positive value of x. The positive unit fractional powers represent roots, which are the inverse to the corresponding integer powers; thus $8^{1/3} = \sqrt[3]{8} = 2$ , since $2^3 = 8$ Note: Power notation is not used for zero, since division by zero is undefined.
<b>expression</b> (KS2)	A mathematical form expressed symbolically. Examples: $7 + 3$ ; $a^2 + b^2$ .
<b>factor</b> (KS2)	When a number, or polynomial in algebra, can be expressed as the product of two numbers or polynomials, these are factors of the first. Examples: 1, 2, 3, 4, 6 and 12 are all factors of 12 because $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ : $(x - 1)$ and $(x + 4)$ are factors of $(x^2 + 3x - 4)$ because $(x - 1)(x + 4) = (x^2 + 3x - 4)$
<b>factorise</b> (KS2)	To express a number or a polynomial as the product of its factors. Examples: Factorising 12: $12 = 1 \times 12$ $= 2 \times 6$ $= 3 \times 4$

	<p>The factors of 12 are 1, 2, 3, 4, 6 and 12.  12 may be expressed as a product of its prime factors:  <math>12 = 2 \times 2 \times 3</math>  Factorising <math>x^2 - 4x - 21</math>:  <math>x^2 - 4x - 21 = (x + 3)(x - 7)</math>  The factors of <math>x^2 - 4x - 21</math> are <math>(x + 3)</math> and <math>(x - 7)</math></p>
<b>facts</b> (KS1)	<p>i.e. Multiplication / division/ addition/ subtraction facts. The word 'fact' is related to the four operations and the instant recall of knowledge about the composition of a number.  i.e. an addition fact for 20 could be <math>10+10</math>; a subtraction fact for 20 could be <math>20-9=11</math>. A multiplication fact for 20 could be <math>4 \times 5</math> and a division fact for 20 could be <math>20 \div 5 = 4</math>.</p>
<b>fluency</b> (KS1)	<p>To be mathematically fluent one must have a mix of conceptual understanding, procedural fluency and knowledge of facts to enable you to tackle problems appropriate to your stage of development confidently, accurately and efficiently.</p>
<b>formal written methods</b> (KS2)	<p>Setting out working in columnar form. In multiplication, the formal methods are called short or long multiplication depending on the size of the numbers involved. Similarly, in division the formal processes are called short or long division. See Mathematics Appendix 1 in the 2013 National Curriculum.</p>
<b>(the) four operations</b>	<p>Common shorthand for the four arithmetic operations of addition, subtraction, multiplication and division.</p>
<b>general statement</b> (KS1)	<p>A statement that applies correctly to all relevant cases. e.g. the sum of two odd numbers is an even number.</p>
<b>generalise</b> (KS1)	<p>To formulate a general statement or rule.</p>
<b>highest common factor</b> (HCF) (KS3)	<p>The common factor of two or more numbers which has the highest value.  Example: 16 has factors 1, 2, 4, 8, 16. 24 has factors 1, 2, 3, 4, 6, 8, 12, 24.  56 has factors 1, 2, 4, 7, 8, 14, 28, 56. The common factors of 16, 24 and 56 are 1, 2, 4 and 8. Their highest common factor is 8.</p>
<b>inequality</b> (KS1)	<p>When one number, or quantity, is not equal to another. Statements such as <math>a \neq b</math>, <math>a &lt; b</math>, <math>a \leq b</math>, <math>a &gt; b</math> or <math>a \geq b</math> are inequalities.  The inequality signs in use are:  <math>\neq</math> means 'not equal to'; <math>A \neq B</math> means 'A is not equal to B'  <math>&lt;</math> means 'less than'; <math>A &lt; B</math> means 'A is less than B'  <math>&gt;</math> means 'greater than'; <math>A &gt; B</math> means 'A is greater than B'  <math>\leq</math> means 'less than or equal to';  <math>A \leq B</math> means 'A is less than or equal to B'  <math>\geq</math> means 'greater than or equal to';  <math>A \geq B</math> means 'A is greater than or equal to B'</p>
<b>integer</b> (KS2)	<p>Any of the positive or negative whole numbers and zero. Example: <math>\dots -2, -1, 0, +1, +2 \dots</math>  The integers form an infinite set; there is no greatest or least integer.</p>
<b>inverse operations</b> (KS1)	<p>Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. <math>5 + 6 - 6 = 5</math>. Multiplication and division are inverse operations e.g. <math>6 \times 10 \div 10 = 6</math>. Squaring and taking the square root are inverse to each other: <math>\sqrt{x^2} = (\sqrt{x})^2 = x</math>; similarly, with cube and cube root, and any integer power <math>n</math> and <math>n</math>th root. Some operations, such as reflection in the x-axis, or 'subtract from 10' are self-inverse i.e. they are inverses of themselves</p>
<b>irrational number</b> (KS3)	<p>A number that is not an integer and cannot be expressed as a common fraction with a non-zero denominator. Examples: <math>\sqrt{3}</math> and <math>\pi</math>.  Real irrational numbers, when expressed as decimals, are infinite, non-recurring decimals.</p>
<b>least common multiple</b> (LCM) (KS3)	<p>The common multiple of two or more numbers, which has the least value. Example: 3 has multiples 3, 6, 9, 12, 15, 18, 21, 24 ..., 4 has multiples 4, 8, 12, 16, 20, 24 ... and 6 has multiples 6, 12, 18, 24, 30 .... The common multiples of 3, 4 and 6 include 12, 24 and 36. The least common multiple of 3, 4 and 6 is 12.</p>
<b>long division</b> (KS2)	<p>A columnar algorithm for division by more than a single digit, most easily described with</p>

	<div>432 ÷ 15 becomes</div> <div><div><div>28.8</div><div>15</div><div>432.0</div></div><div><div>30</div><div>132</div><div>120</div><div>120</div><div>0</div></div></div> <div>an example: Answer: 28.8</div>																																
long multiplication (KS2)	<div>A columnar algorithm for performing multiplication by more than a single digit, again best illustrated by an example:</div> <div>124 × 26 becomes</div> <div><div><div>124</div><div>× 26</div><div>744</div><div>2480</div><div>3224</div></div><div>11</div></div> <div>Answer: 3224</div> <div>(Example taken from Appendix 1 of the Primary National Curriculum for Mathematics)</div>																																
mental calculation (KS1)	Referring to calculations that are largely carried out mentally, but may be supported with a few simple written jottings.																																
minus (KS1)	A name for the symbol −, representing the operation of subtraction.																																
missing number problems (KS1)	A problem of the type 7 = □ − 9 often used as an introduction to algebra.																																
multiple (KS1)	<div>For any integers a and b, a is a multiple of b if a third integer c exists so that a = bc</div> <div>Example: 14, 49 and 70 are all multiples of 7 because 14 = 7 × 2, 49 = 7 × 7 and 70 = 7 × 10.. -21 is also a multiple of 7 since -21 = 7 × -3.</div>																																
multiplicand (KS1)	<div>A number to be multiplied by another.</div> <div>e.g. in 5 × 3, 5 is the multiplicand as it is the number to be multiplied by 3.</div> <div>See also Addend, subtrahend and dividend.</div>																																
multiplication (KS1)	<div>Multiplication (often denoted by the symbol "×") is the mathematical operation of scaling one number by another. It is one of the four binary operations in arithmetic (the others being addition, subtraction and division).</div> <div>Because the result of scaling by whole numbers can be thought of as consisting of some number of copies of the original, whole-number products greater than 1 can be computed by repeated addition; for example, 3 multiplied by 4 (often said as "3 times 4") can be calculated by adding 4 copies of 3 together:</div> <div>3 × 4 = 3 + 3 + 3 + 3 = 12</div> <div>Here 3 and 4 are the "factors" and 12 is the "product".</div> <div>Multiplication is the inverse operation of division, and it follows that</div> <div>7 ÷ 5 × 5 = 7</div> <div>Multiplication is commutative, associative and distributive over addition or subtraction.</div>																																
multiplication table (KS1)	<div>An array setting out sets of numbers that multiply together to form the entries in the array, for example</div> <table><tr><td>Multipliers</td><td>1</td><td>2</td><td>3</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td></tr><tr><td>2</td><td>2</td><td>4</td><td>6</td></tr><tr><td>3</td><td>3</td><td>6</td><td>9</td></tr><tr><td>4</td><td>4</td><td>8</td><td>12</td></tr><tr><td>5</td><td>5</td><td>10</td><td>15</td></tr><tr><td>6</td><td>6</td><td>12</td><td>18</td></tr><tr><td>7</td><td>7</td><td>14</td><td>21</td></tr></table>	Multipliers	1	2	3	1	1	2	3	2	2	4	6	3	3	6	9	4	4	8	12	5	5	10	15	6	6	12	18	7	7	14	21
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4	4	8	12																														
5	5	10	15																														
6	6	12	18																														
7	7	14	21																														
multiplicative reasoning (KS2)	Multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts.																																

	<p>For example, from this: 3 bags of sweets, 8 sweets in each bag. How many sweets? To this and beyond: Julie bought a dress in a sale for £49.95 after it was reduced by 30%. How much would she have paid before the sale?</p>
<b>multiply</b> (KS1)	Carry out the process of multiplication.
<b>notation</b> (KS1)	A convention for recording mathematical ideas. Examples: Money is recorded using decimal notation e.g. £2.50 Other examples of mathematical notation include $a + a = 2a$ , $y = f(x)$ and $n \times n \times n = n^3$ ,
<b>number bond</b> (KS1)	A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10.
<b>number line</b> (KS1)	A line where numbers are represented by points upon it.
<b>number sentence</b> (KS1)	A mathematical sentence involving numbers. Examples: $3 + 6 = 9$ and $9 > 3$
<b>number track</b> (KS1)	A numbered track along which counters might be moved. The number in a region represents the number of single moves from the start.
<b>odd number</b> (KS2)	An integer that has a remainder of 1 when divided by 2.
<b>order of magnitude</b> (KS2)	<p>The approximate size, often given as a power of 10. Example of an order of magnitude calculation: <math>95 \times 1603 \div 49 \approx 102 \times 16 \times 102 \div (5 \times 101) \approx 3 \times 103</math></p>
<b>order of operation</b> (KS2)	<p>This refers to the order in which different mathematical operations are applied in a calculation. Without an agreed order an expression such as <math>2 + 3 \times 4</math> could have two possible values: <math>5 \times 4 = 20</math> (if the operation of addition is applied first) <math>2 + 12 = 14</math> (if the operation of multiplication is applied first) The agreed order of operations is that:</p> <ul style="list-style-type: none"> <li>• Powers or indices take precedent over multiplication or division – <math>2 \times 3^2 = 18</math> not 25;</li> <li>• Multiplication or division takes precedent over addition and subtraction – <math>2 + 3 \times 4 = 14</math> not 20</li> <li>• If brackets are present, the operation contained therein always takes precedent over all others – <math>(2 + 3) \times 4 = 20</math></li> </ul> <p>This convention is often encapsulated in the mnemonic BODMAS or BIDMAS: Brackets Orders / Indices (powers) Division &amp; Multiplication Addition &amp; Subtraction</p>
<b>partition</b> (KS1)	<ol style="list-style-type: none"> <li>1. To separate a set into subsets.</li> <li>2. To split a number into component parts. Example: the two-digit number 38 can be partitioned into <math>30 + 8</math> or <math>19 + 19</math>.</li> <li>3. A model of division. Example: <math>21 \div 7</math> is treated as 'how many sevens in 21?'</li> </ol>
<b>pictorial representations</b> (KS1)	Pictorial representations enable learners to use pictures and images to represent the structure of a mathematical concept. The pictorial representation may build on the familiarity with concrete objects. E.g. a square to represent a Dienes 'flat' (representation of the number 100). Pupils may interpret pictorial representations provided to them or create a pictorial representation themselves to help solve a mathematical problem.
<b>plus</b> (KS1)	A name for the symbol +, representing the operation of addition.
<b>power (of ten)</b> (KS2)	<ol style="list-style-type: none"> <li>1. 100 (i.e. <math>10^2</math> or <math>10 \times 10</math>) is the second power of 10, 1000 (i.e. <math>10^3</math> or <math>10 \times 10 \times 10</math>) is the third power of 10 etc. Powers of other numbers are defined in the same way. Example: 2 (<math>2^1</math>), 4 (<math>2^2</math>), 8 (<math>2^3</math>), 16 (<math>2^4</math>) etc are powers of 2.</li> <li>2. A fractional power represents a root. Example: <math>x^{1/2} = \sqrt{x}</math></li> <li>3. A negative power represents the reciprocal. Example: <math>x^{-1} = 1/x</math></li> <li>4. By convention any number or variable to the power 0 equals 1. i.e. <math>x^0 = 1</math></li> </ol>
<b>prime factor</b> (KS2)	The factors of a number that are prime. Example: 2 and 3 are the prime factors of 12 ( $12 = 2 \times 2 \times 3$ ). See also factor.

<b>prime factor decomposition</b> (KS2)	The process of expressing a number as the product of factors that are prime numbers. Example: $24 = 2 \times 2 \times 2 \times 3$ or $2^3 \times 3$ . Every positive integer has a unique set of prime factors.
<b>prime number</b> (KS2)	A whole number greater than 1 that has exactly two factors, itself and 1. Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1).
<b>priority of operations</b> (KS2)	Generally, multiplication and division are done before addition and subtraction, but this can be ambiguous, so brackets are used to indicate calculations that must be done before the remainder of the operations are carried out. See order of operation
<b>product</b> (KS1)	The result of multiplying one number by another. Example: The product of 2 and 3 is 6 since $2 \times 3 = 6$ .
<b>quotient</b> (KS2)	The result of a division. Example: $46 \div 3 = 15\frac{1}{3}$ and $15\frac{1}{3}$ is the quotient of 46 by 3. Where the operation of division is applied to the set of integers, and the result expressed in integers, for example $46 \div 3 = 15$ remainder 1 then 15 is the quotient of 46 by 3 and 1 is the remainder.
<b>recurring decimal</b> (KS2)	A decimal fraction with an infinitely repeating digit or group of digits. Example: The fraction $\frac{1}{3}$ is the decimal 0.33333 ..., referred to as nought point three recurring and may be written as 0.3 (with a dot over the three). Where a block of numbers is repeated indefinitely, a dot is written over the first and last digit in the block e.g. $\frac{1}{7} = 0.\dot{1}42857\dot{}$
<b>remainder</b> (KS2)	In the context of division requiring a whole number answer (quotient), the amount remaining after the operation. Example: 29 divided by 7 = 4 remainder 1.
<b>repeated addition</b> (KS1)	The process of repeatedly adding the same number or amount. One model for multiplication. Example $5 + 5 + 5 + 5 = 5 \times 4$ .
<b>repeated subtraction</b> (KS1)	The process of repeatedly subtracting the same number or amount. One model for division. Example $35 - 5 - 5 - 5 - 5 - 5 - 5 - 5 = 0$ so $35 \div 5 = 7$ remainder 0.
<b>representation</b> (KS2)	The word 'representation' is used in the curriculum to refer to a particular form in which the mathematics is presented, so for example a quadratic function could be expressed algebraically or presented as a graph; a quadratic expression could be shown as two linear factors multiplied together or the multiplication could be expanded out; a probability distribution could be presented in a table or represented as a histogram, and so on. Very often, the use of an alternative representation can shed new light on a problem. An array is a useful representation for multiplication and division which helps to see the inverse relationship between the two. The Bar Model is a useful representation of for many numerical problems. E.g. Tom has 12 sweets and Dini has 5. How many more sweets does Tom have than Dini? 
<b>scale (verb)</b> (KS2)	To enlarge or reduce a number, quantity or measurement by a given amount (called a scale factor). e.g. to have 3 times the number of people in a room than before; to find a quarter of a length of ribbon; to find 75% of a sum of money.
<b>share (equally)</b> (KS1)	Sections of this page that are currently empty will be filled over the coming weeks. One model for the process of division.
<b>short division</b> (KS2)	A compact written method of division. Example: $\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \\ \underline{11} \phantom{0} \\ 49 \phantom{0} \\ \underline{44} \phantom{0} \\ 56 \\ \underline{55} \\ 1 \end{array}$ <p>Answer: <math>45 \frac{1}{11}</math> (Example taken from Appendix 1 of the Primary National Curriculum for Mathematics)</p>
<b>short multiplication</b> (KS2)	Essentially, simple multiplication by a one digit number, with the working set out in columns. $\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \phantom{2394} 21 \\ \hline \end{array}$ <p>Answer: 2394 (Example taken from Appendix 1 of the Primary National Curriculum for Mathematics)</p>

<b>sign</b> (KS1)	A symbol used to denote an operation. Examples: addition sign +, subtraction sign –, multiplication sign ×, division sign ÷, equals sign = etc. In the case of directed numbers, the positive + or negative – sign indicates the direction in which the number is located from the origin along the number line.
<b>square</b> (KS1)	1. A quadrilateral with four equal sides and four right angles. 2. The square of a number is the product of the number and itself. Example: the square of 5 is 25. This is written $5^2 = 25$ and read as five squared is equal to twenty-five. See also square number and square root.
<b>square number</b> (KS2)	A number that can be expressed as the product of two equal numbers. Example $36 = 6 \times 6$ and so 36 is a square number or “6 squared”. A square number can be represented by dots in a square array.
<b>square root</b> (KS3)	A number whose square is equal to a given number. Example: one square root of 25 is 5 since $5^2 = 25$ . The square root of 25 is recorded as $\sqrt{25} = 5$ . However, as well as a positive square root, 25 has a negative square root, since $(-5)^2 = 25$ .
<b>subtract</b> (KS1)	Carry out the process of subtraction
<b>subtraction</b> (KS1)	The inverse operation to addition. Finding the difference when comparing magnitude. Take away.
<b>subtraction by decomposition</b> (KS2)	A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. In this method the number to be subtracted from (the <b>minuend</b> ) is re-partitioned, if necessary, in order that each digit of the number to be subtracted (the <b>subtrahend</b> ) is smaller than its corresponding digit in the minuend. e.g. in $719 - 297$ , only the digits in the hundreds and the ones columns are bigger in the minuend than the subtrahend. By regrouping 719 into 6 hundreds, 11 tens and 9 ones each separate subtraction can be performed simply, i.e.: $9 - 7$ , $11 \text{ (tens)} - 9 \text{ (tens)}$ and $6 \text{ (hundreds)} - 2 \text{ (hundreds)}$ . $\begin{array}{r} 6 \text{ } 11 \text{ } 9 \\ - 2 \text{ } 9 \text{ } 7 \\ \hline 4 \text{ } 2 \text{ } 2 \end{array}$
<b>subtraction by equal addition</b>	A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. This method relies on the understanding that adding the same quantity to both the <b>minuend</b> and the <b>subtrahend</b> retains the same difference. This is a useful technique when a digit in the subtrahend is larger than its corresponding digit in the minuend. E.g. in the example below, $7 > 2$ , therefore 10 has been added to the 2 (in the ones place) of the minuend to make 12 (ones) and also added to the 5 (tens) of the subtrahend to make 60 (or 6 tens) before the first step of the calculation can be completed. Similarly 100 has been added to the 3 (tens) of the minuend to make 13 (tens) and also added to the 4 (hundreds) of the subtrahend to make 5 (hundred). 932-457 becomes $\begin{array}{r} 1 \quad 1 \\ 9 \quad 3 \quad 2 \\ - 4 \quad 5 \quad 7 \\ \hline 5 \quad 6 \\ 4 \quad 7 \quad 5 \end{array}$ Example taken from Appendix 1 of the Primary National Curriculum for Mathematics
<b>subtrahend</b> (KS1)	A number to be subtracted from another. See also <b>Addend</b> , dividend and multiplicand.
<b>sum</b> (KS1)	The result of one or more additions.
<b>symbol</b> (KS1)	A letter, numeral or other mark that represents a number, an operation or another mathematical idea. Example: L (Roman symbol for fifty), > (is greater than).
<b>take away</b> (KS1)	1. Subtraction as reduction 2. Remove a number of items from a set.
<b>terminating decimal</b> (KS2)	A decimal fraction that has a finite number of digits. Example: 0.125 is a terminating decimal. In contrast $\frac{1}{3}$ is a recurring decimal fraction. All terminating decimals can be expressed as fractions in which the denominator is a

	multiple of 2 or 5.
<b>total</b> (KS1)	<ol style="list-style-type: none"> <li>1. The aggregate. Example: the total population - all in the population.</li> <li>2. The sum found by adding.</li> </ol>
<b>zero</b> (KS1)	<ol style="list-style-type: none"> <li>1. Nought or nothing; zero is the only number that is neither positive nor negative.</li> <li>2. Zero is needed to complete the number system. In our system of numbers :  <math>a - a = 0</math> for any number <math>a</math>.  <math>a + (-a) = 0</math> for any number <math>a</math>;  <math>a + 0 = 0 + a = a</math> for any number <math>a</math>;  <math>a - 0 = a</math> for any number <math>a</math>;  <math>a \times 0 = 0 \times a = 0</math> for any number <math>a</math>;  division by zero is not defined as it leads to inconsistency.</li> <li>3. In a place value system, a place-holder. Example: 105.</li> <li>4. The cardinal number of an empty set.</li> </ol>